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FRANCISCI M. DE REGI

CLER. REGUL. S. PAULLI,

ATQUE IN MEDIOL. UNIVERSITATE S. ALEX.
MATHESEOS PROFESSORIS

THEOREMATA,

IN QUIBUS

PLURES CIRCULI AD POLYGONA, SPHÆRÆ AD SOLIDA INSCRIPTA, ET CORPORUM REGULARIUM INTER SE PROPORTIONES DEMONSTRANTUR; ITEM VARIA AD RECTANGULORUM SUMMAS, AC DIFFERENTIAS IN CIRCULO, ET AD TRISECTIONEM ARCUS PERTINENTIA, NONNULLÆQUE ALIÆ VERITATES GEOMETRICÆ CONTINENTUR.

MEDIOLANI)(MDCCLVII.

EX TYPOGRAPHIA SOCIETATIS PALATINÆ
IN REGIA CURIA.
SUPERIORUM FACULTATE.

6-25-27 N.G.5.

CL. MATHEMATICO
CAROLO JOSEPHO
MERULÆ

EX MEDIOLANENSI ARCHITECTORUM COLLEGIO,
GENERALI REGIORUM FINIUM,
ATQUE SENATUS EXCELLENTISSIMI
ARCHITECTO

FRANCISCUS MARIA DE' REGI
CLER. REGUL. S. PAULLI
S. D.

EO *tempore, quo ſtatui nonnulla Opuſcula mea typis committere a me tunc elucubrata, quando id mihi præſtare non prohibuit aſſidua tum publice, tum privatim in Mathematicis Scientiis eos erudiendi occupatio, qui hac de cauſſa ad me conveniunt: illo ipſo etiam decrevi eadem ad Te mittere, Teque exorare, quemadmodum facio, ut*

ea, veluti ſi a Te fuiſſent compoſita, reciperes. Non levis autem ad id me cauſſa impulit; nam hoc mihi ſemper perſuaſum fuit, quodcumque opus illi potiſſimum eſſe tradendum, qui, utpote rerum, quæ in opere continentur, peritiſſimus, de eodem etiam certam, ac tutam poteſt proferre ſententiam. Quamobrem cui niſi tibi hæc mea opuſcula dabo? Tu ille es, qui a primis juventutis tuæ annis uſque ad hanc diem ætate jam provectus univerſas Scientiæ Mathematicæ partes rara diſcretione coluiſti; quemque admirantur, quotquot inviſendi gratia quavis diei hora Te adeunt, impigre verſantem in tua Bibliotheca copioſis, atque ſelectis libris, potiſſimum mathematicis, inſtructa. Tu ille es, qui, cum univerſali polleas Theoria, cuique Te de Rebus Mathematicis interroganti vel prompte, ac dilucide ſatisfacis, vel certos indicas fontes, ex quibus ſingula poſſint depromi. Cumque perfectiſſimæ Theoriæ præcipuus eſſe debeat fructus æque perfecta, atque accuratiſſima Praxis, Tu in quocumque ejuſdem genere excellis. Anteactis enim annis, quamquam graviſſimis tam privatis, quam publicis eſſes diſtentus curis, ut genio tuo indulge-

res,

res, modo unam, modo alteram Machinam manu tua ſolebas exſtruere, modo vitra polire, modo inſtrumenta mathematica cujuſcumque generis perficere, idque tanta perfectione præſtabas quanta teſtatur in ſua Specula Domeſtica (a) *Clariſſimus Marinonus Mathematicus Cæſareus elaboratum fuiſſe Quadrantem Aſtronomicum dono eidem, ut Viro tibi amiciſſimo, datum. Tu ille es, quem, cum in rebus & ſpeculandis, & exercendis expertiſſimum, quicumque has callent ſcientias, agnoverint, ad præcipua ideo munera ſelegerunt. Nam, cum de Reno Bononienſi dirigendo ageretur, qua de cauſſa prima Mathematicorum lumina illuc convenerant, inter quos Grandius, Marinonus, Zendrinus, Corradus, Manfredii, ex Urbe noſtra evocatus es, ut Tu etiam conſuli poſſes; cum vero de Ticini curſu moderando variæ eſſent Celebrio-*

rum

(*a*) *Lib. 2. de Organico Apparatu Aſtronomico Paragr. 3. pag.* 112. Hunc Vir eximius, diu intima mihi amicitia conjunctus, Carolus Joſephus Merula Mediolanenſis Collegiatus, & ad integri Status confinia deputatus Geometra, Civilis Architecturæ, Hydrometriæ, Dioptricæ, aliarumque partium Matheſeos apprimè doctus, a domeſtico Mechanico ſibi conſtrui curavit, ipſeque in gradus, & minuta peculiari artificio diviſit, tubos teleſcopicos lentibus a ſe politis inſtruxit, deinde ad uſum meæ Speculæ, inſculpto ſuo nomine, mihi donavit.

rum Hominum ſententia, tuam ipſe, & verbo, & ſcriptis ita firmaſti, ut probabilior haberetur; in Novocomenſi autem Agro ita alvei omnes tuo conſilio, & arte fuerunt expurgati, & aquæ diductæ, ut deinceps Comenſis Civitas exundationibus multo minus fuerit obnoxia. At, quid ego pergam per ſingula, cum univerſi hi noſtri Cives, quanti ſis, abunde teſtentur? Tibi enim præcipue committunt, ſi qua ardua ſunt vel in ædificiis conſtruendis, vel in Machinis moliendis, vel in aquis derivandis, vel in qualibet Mathematica Re perficienda. Quare quem alium, præter Te, velim operum meorum Judicem? Quamquam hæc tecum alia etiam de cauſſa erant communicanda; abſtractas ſiquidem, atque geometricas tantum continent propoſitiones hæc mea Opuſcula; nullam ego antehac inauditam Mechanicæ legem, nullam uſque modo incognitam naturæ vim, aut ejuſdem motum, nullum Satellitem, aut ſidus novum detego, nihil, inquam uno verbo, eorum, quæ Eruditorum Hominum admirationem, ac deſiderium excitare poſſint. Cum itaque ea Tu mihi neceſſitudine ſis conjunctus, qua nulla magis, auſim dicere, familiaris,

& in-

& intima, quamque teſtatus es ſemper humaniſſimis quibuſcumque officiis tuis, præſertim vero cum dilectiſſimum tuum Filium, illumque unicum mihi Matheſi imbuendum tradidiſti, patienter, atque æquo animo leges, quod fortaſſe alii deſpicient. Accipe igitur hoc, quidquid eſt, amicitiæ, ac veteris conſuetudinis noſtræ veluti pignus; ſi enim tibi uni probatum id fuerit, ſatis emolumenti ex hiſce lucubrationibus meis me percepiſſe judicabo. Vale.

DON PAULLUS PHILIPPUS PREMOLI

Congregationis Clericorum Regularium S. Paulli Præpoſitus Generalis.

QUum Librum, cui Titulus eſt = *Theoremata, in quibus plures Circuli ad Polygona, Sphæræ ad Solida inſcripta, & corporum regularium inter ſe proportiones demonſtrantur; item varia &c.* = a Patre D. Franciſco Maria de' Regi Congregationis noſtræ Presbytero Profeſſo compoſitum, duo ejuſdem Congregationis noſtræ eruditi Viri, quibus id commiſimus, accurata lectione, & gravi Judicio recognoverint, & poſſe in lucem edi probaverint; nos ut Typis mandetur, quantum in nobis eſt, facultatem facimus. In quorum fidem has fieri, ſigilloque noſtro muniri juſſimus. Dat. Mediolani die 23. Menſ. Feb. anno 1757. ex Collegio noſtro SS. Apoſt. Paulli, & Barnabæ.

D. PAULLUS PHILIPPUS PREMOLI
PRÆPOSITUS GENERALIS.

Locus ✠ Sigilli.

Don Carolus Bonfigli Cancellarius.

Lib. Com. Magione
3-1-28
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THEOREMATA,

IN QUIBUS

Plures Circuli ad Polygona, Sphæræ ad Solida inſcripta; & corporum regularium inter ſe proportiones demonſtrantur.

ANNO quadrageſimo nono ſupra milleſimum, & ſeptingenteſimum, cum ſuum de Etruſca Eloquentia opus elegantiſſimum conſcriberet Congregationis noſtræ Vir doctiſſimus Salvator Corticellius, aliquid a me elucubratum quæſierat eidem inſerendum. Videram autem paullo ante Eximii Viri Franciſci Zanotti duo illa univerſalia, ac nunquam ſatis commendata Theoremata de proportione figurarum planarum ad circulum, & ſolidarum ad ſphæram Inſcriptam, de quibus litterariæ Ephemerides Florentinæ mentionem fecerant. Quamobrem mecum ipſe cogitare cœpi, ſi quid ſimile etiam eveniret in circulis reſpectu ad figuras rectilineas, ac in ſphæris reſpectu ad ſolida, quæ iiſdem inſcribuntur. Tum igitur, ut Hominis, quem plurimi ſemper feci, voluntati obtemperarem, tum ut deſiderium meum, quo jam tene-

bar, explerem, hæc ipſa ad trutinam revocavi: quod ut rite peragerem opportunum eſſe maximè arbitratus ſum iiſdem Zanotti Theorematibus uti. Nec male res ceſſit; vix enim circulo polygonum, & polygono circulum, ſphæræ ſolidum, & ſolido ſphæram inſcripſi, ut ſtatim animadverti has figuras conſtantem inter ſe proportionem habere. Sex itaque Theoremata inveni; principiis ex Zanotti propoſitionibus deductis eadem demonſtravi: atque, ut eorum uſum, & utilitatem indicarem, plures Veterum propoſitiones longis, implicatiſque ratiociniis olim demonſtratas, ex brevibus, perſpicuiſque demonſtrationibus erutas tradidi. Ubi vero opus abſolvi, ea ad Corticellium miſi, ut de illis tum per ſe ipſum, tum per alios examen inſtitueret, &, quidquid ſentiret, decerneret. Theoremata conſideravit inter ceteros Zanottus, nec cenſuit contemnenda: Modum interea cogitabat Corticellius, quo eadem in opus ſuum induceret: ſed cum nimium ab inſtituto ſuo digrediendum ſibi videret, ſi omnia complecteretur, unum tantum ſibi aſſumpſit Theorema; reliqua vero, ut in ſymbolis ſuis imprimenda curaret eruditiſſimus Vir, ac de Republica litteraria optime meritus Præpoſitus Antonius Gorius, effecit. Cum vero paullo poſt ea perlegiſſet Comes Gregorius Caſalius in Bononienſi Inſtituto Architecturæ Militaris Profeſſor, qui, cum in Phyſicis, æque ac Mechanicis Rebus ſit apprime verſatus, ut ſoler-

lertiſſimus quoque Mathematicus merito eſt laudandus, præſertim ob Tetrahedri, Hexahedri, & Octahedri inſcriptiones incomparabili facilitate, mira brevitate, ac perſpicuitate demonſtratas, atque ea probaviſſet, ac ſupra meritum commendaſſet, animum mihi addidit, ut iterum ea typis committerem una cum pluribus aliis Theorematis, quorum nonnulla Clariſſimus Zanottus ſibi communicata non improbaverat, ſperans fore, ut omnia apud utrumque eandem inirent ſortem, nec fortaſſe ceteris eſſent injucunda.

THEOREMA I.

Ambitus cujuslibet figuræ planæ rectilineæ, cui circumſcribi, & inſcribi poteſt circulus, eſt ad circumferentiam circuli circumſcripti, ut rectangulum ex perimetro in radium circuli inſcripti, ſeu ut dupla area figuræ rectilineæ inſcriptæ ad rectangulum ex peripheria minori in radium majoris.

Demonſtratio.

Sit circumferentia circuli circumſcripti $= C$
— — — — — — — inſcripti $= c$
— — perimeter figuræ rectilineæ $= p$
— — — — radius circuli majoris $= R$
— — — — — — minoris $= r$

erit $C : c = R : r$;

 ſed

ſed ex Th.I. Cl. Zanotti $c:p=\frac{cr}{2}:\frac{pr}{2}$, & $Cc:cp=\frac{Rcr}{2}:\frac{pr^2}{2}$;

ergo $C:p=Rc:pr$. Q. e. d.

Corollarium I.

Si triangulo æquilatero circumſcribatur, & inſcribatur circulus, erit $r=\frac{R}{2}$, ac proinde $c=\frac{C}{2}$; cum igitur ſit $C:p=Rc:pr$, erit etiam $C:p=\frac{CR}{2}:pr$; ſed $\frac{CR}{2}$ eſt area circuli circumſcripti, pr dupla area trianguli æquilateri, ſeu area hexagoni eidem circulo inſcripti; ergo circumferentia circuli triangulo æquilatero circumſcripti eſt ad perimetrum trianguli inſcripti, ut area ejuſdem circuli ad aream hexagoni.

Corollarium II.

Hinc erit ſuperficies priſmatis, cui inſcribi, & circumſcribi poteſt cylindrus, ad ſuperficiem cylindri circumſcripti, ſecluſis baſibus, ut dupla baſis priſmatis ad rectangulum ex radio baſis cylindri circumſcripti in peripheriam baſis cylindri inſcripti.

Co-

Corollarium III.

Quod ſi priſma triangulare cylindro inſcriptum baſim habet æquilateram, erit ſuperficies cylindri ad ſuperficiem priſmatis, non conſideratis baſibus, ut baſis cylindri ad baſim priſmatis regularis, cujus baſis ſit hexagonum in eadem cylindri baſi inſcriptum per Cor. I.

THEOREMA II.

Area cujuslibet figuræ planæ rectilineæ, cui inſcribi, & circumſcribi poteſt circulus, eſt ad aream circuli circumſcripti, ut parallelepipedum, cujus altitudo eſt perimeter, & baſis quadratum diametri circuli inſcripti, ad parallelepipedum, cujus altitudo eſt circumferentia minor, & baſis quadratum diametri majoris.

Demonſtratio.

Sit area circuli circumſcripti $= A$
— — — — — inſcripti $= a$
— — — figuræ inſcriptæ $= \mathrm{a}$
circumferentia circuli circumſc. $= C$
— — — — — inſcripti $= c$
— — — perimeter figuræ $= p$
— diameter circuli majoris $= D$
— — — — — minoris $= d$
erit $A : a = D^2 : d^2$;

ſed

ſed per Theor. I. Zanotti $a : \mathrm{a} = c : p$;
& $\mathrm{A}a : a\mathrm{a} = D^2 c : d^2 p$;
ergo $A : \mathrm{a} = D^2 c : d^2 p$. Q. e. d.

Corollarium I.

Si triangulo æquilatero inſcribatur, & circumſcribatur circulus, cum tunc ſit $d = \frac{D}{2}$, erit etiam $c = \frac{C}{2}$; quare erit $A : \mathrm{a} = \frac{CD^2}{2} : \frac{pD^2}{4} = 2CD^2 : pD^2 = 2C : p$. Area igitur circuli circumſcripti ad aream trianguli æquilateri inſcripti duplam habet rationem ea, quam habet circumferentia circuli circumſcripti ad perimetrum trianguli; ſed circumferentia eſt ad perimetrum, ut area circuli circumſcripti ad aream hexagoni inſcripti per Cor. I. Theor. I.; ergo area circuli circumſcripti eſt ad aream trianguli, ut dupla area ejuſdem circuli circumſcripti ad aream hexagoni inſcripti; ex quo infertur aream hexagoni duplam eſſe area trianguli æquilateri.

Corollarium II.

Quod ſi circulus inſcribatur, & circumſcribatur quadrato; tunc $d = \frac{D}{\sqrt{2}}$, ac proinde $cD^2 = \frac{CD^2}{\sqrt{2}}$; ergo $A : \mathrm{a} = \frac{CD^2}{\sqrt{2}} : \frac{D^2 p}{2}$; ſed $p = \frac{4D}{\sqrt{2}}$; ergo $A : \mathrm{a} = \frac{CD^2}{\sqrt{2}} : \frac{4D^3}{2\sqrt{2}}$ $= CD^2 : 2D^3 = C : \frac{D}{2}$,

Corol-

Corollarium III.

Vel hexagono; quoniam $D^2c = CdD$, erit $A : a = CdD : d^2p$; ſed $p = 3D$; ergo $A : a = CdD : 3Dd^2 = C : 3d$. Eſt igitur area circuli majoris ad aream hexagoni inſcripti, ut circumferentia ejuſdem circuli ad perimetrum hexagoni circulo minori inſcripti; ſed per Corol. I. Theor. I. area circuli circumſcripti eſt ad aream hexagoni inſcripti, ut ejuſdem peripheria ad perimetrum trianguli æquilateri; ergo ambitus trianguli æquilateri circulo majori inſcripti adæquat perimetrum hexagoni circulo minori inſcripti.

Corollarium IV.

Hinc ſi circulo inſcribatur hexagonum, & hexagono circulus, & huic aliud hexagonum, atque ita in infinitum, tam hexagonorum perimetri, quam eorundem areæ erunt in proportione continua; nam, ſi cuilibet circulo intelligatur inſcriptum triangulum æquilaterum, & vocetur perimeter hexagoni majoris $=$ E; perimeter ſecundi $= E$; & perimeter tertii $= e$; item ambitus trianguli æquilateri majoris $=$ T, ambitus ſecundi $= T$, & ambitus tertii $= t$, erit E : T $= E : T$; ſed per Corol. præc. T $= E$; ergo E : $E = E : T$; pariter erit $E : T = e : t$; ſed per idem Corol. $T = e$; ergo $E : e = e : t =$ E : E;

ergo

ergo $E : \mathit{E} = \mathit{E} : e$; Sunt igitur perimetri E, *E*, *e* in proportione continua, &, quia areæ ſunt in duplicata ratione perimetrorum, hæ quoque erunt in proportione continua; ſed circulorum circumferentiæ, & triangulorum ambitus ſunt, ut perimetri hexagonorum, quemadmodum eorum areæ, ut areæ hexagonorum; ergo ſunt in proportione continua: imo quæcumque ſint figuræ ſimiles circulis inſcriptæ erunt in proportione continua tam earum perimetri, quam areæ. Suppoſito ergo perimetro hexagoni majoris = 1., eorum ambitus, ſicuti omnium figurarum ſimilium, juxta hanc numerorum ſeriem progredientur $1. \frac{\sqrt{3}}{2} . \frac{3}{4} . \frac{\sqrt{27}}{8} . \frac{9}{16}$ &c. in infinitum, atque eorum areæ juxta hanc aliam ſeriem $1. \frac{3}{4} . \frac{9}{16} . \frac{27}{64} . \frac{81}{256}$. &c. in infinitum:

Corollarium V.

Quod ſi intelligemus ductam eſſe diametrum tum circulos, tum hexagona, tum triangula æquilatera bifariam dividentem, atque circa hanc revolvi concipiemus ſive ſemicirculos, ſive ſemihexagona, ſive ſemitriangula, ex primis orientur ſphæræ, ex ſecundis corpora compoſita ex duobus conis, & cylindro, ex tertiis denique coni, quorum corporum ſuperficies erunt, ut areæ figurarum ea producentium, cum debeant eſſe in duplicata ratione laterum, & ſoliditates erunt, ut cubi peri-

perimetrorum, cum ſint in triplicata ratione laterum. Quare ſoliditates hanc numerorum ſeriem conſtituent $1 \cdot \frac{3\sqrt{3}}{8} \cdot \frac{27}{64} \cdot \frac{81\sqrt{3}}{512}$ &c. in infinitum; quod æque verum erit de ſoliditatibus omnium ſimilium corporum, quæ his inſcribi poſſunt ſphæris, ut patet.

THEOREMA III.

Superficies corporis, cui inſcribi, & circumſcribi poteſt ſphæra, eſt ad ſuperficiem ſphæræ circumſcriptæ in ratione compoſita rationis ſuperficiei corporis ad ſuperficiem ſphæræ inſcriptæ, & ejus, quam habet quadratum diametri ſphæræ minoris ad quadratum diametri ſphæræ majoris.

Demonſtratio.

Sit ſuperficies ſphæræ circumſcriptæ $= A$
— — — — — — inſcriptæ $= a$
— — — — corporis inſcripti $= \mathrm{a}$
— diameter ſphæræ circumſcriptæ $= D$
— — — — — — inſcriptæ $= d$

erit $A : a = D^2 : d^2$;

ſed per Theorema II. Zanotti $a : \mathrm{a} = \frac{da}{6} : \frac{d\mathrm{a}}{6}$;

quare $Aa : a\mathrm{a} = \frac{daD^2}{6} : \frac{\mathrm{a}d^3}{6}$;

ergo $A : \mathrm{a} = aD^2 : \mathrm{a}d^2$. Q. e. d.

Corollarium I.

Si corpus, cui inſcribitur, & circumſcribitur ſphæra, ſit conus æquilaterus, cum in hoc caſu ſit $d = \frac{D}{2}$ erit $A : \mathrm{a} = aD^2 : \frac{\mathrm{a}D^2}{4} = a : \frac{\mathrm{a}}{4}$; ſed $a : \mathrm{a} = 4 : 9$, ac proinde $a = \frac{4\mathrm{a}}{9}$; ergo $A : \mathrm{a} = \frac{4\mathrm{a}}{9} : \frac{\mathrm{a}}{4} = 16 : 9$.

Corollarium II.

Sed ſi eſt cylindrus quadratus; tunc, cum ſit $D^2 = 2d^2$, & $a = \frac{2\mathrm{a}}{3}$, erit $A : \mathrm{a} = \frac{2\mathrm{a}D^2}{3} : \mathrm{a}d^2 = \frac{2D^2}{3} : d^2 = \frac{4d^2}{3} : d^2 = 4 : 3$. Quod ſi cylindri ſuperficies conſiderabitur, ſecluſis baſibus, quæ adæquant tertiam ejus partem, erit $A : \frac{2\mathrm{a}}{3} = \frac{2\mathrm{a}D^2}{3} : \frac{2\mathrm{a}d^2}{3} = D^2 : d^2 = 2 : 1$.

Corollarium III.

Hinc, cum ſit ſuperficies ſphæræ ad ſuperficiem cylindri quadrati inſcripti, ut $4 : 3$, & eadem ſuperficies ſphæræ ad ſuperficiem coni æquilateri inſcripti, ut $16 : 9$, ſeu, ut $4 : 9$, erit ſuperficies cylindri ad illam coni, ut $3 : \frac{9}{4}$; ſed $3 : \frac{9}{4} = 4 : 3$; ergo ſuperficies ſphæræ, cylin-

dri

dri quadrati, & coni æquilateri, quæ ſunt in ſphæra inſcripta, eandem continuant proportionem.

Corollarium IV.

Cumque ſit ſuperficies ſphæræ circumſcriptæ ad ſuperficiem integram cylindri quadrati, ut 4 : 3, & illa ad hanc ſine baſibus, ut 2 : 1, ſeu, ut 4 : 2, erunt ſuperficies ſphæræ, ſuperficies cylindri quadrati integra, & eadem ſine baſibus in proportione arithmetica continua, ideſt, ut 4 : 3 : 2.

Corollarium V.

Inſcripta, circumſcriptaque ſphæra quadrato rhombo conico, habetur $d = \frac{D}{\sqrt{2}}$, & $a : \mathrm{a} = d : D$; ergo $A : \mathrm{a} = dD^2 : Dd^2 = D : d = \sqrt{2} : 1$, ideſt, ſuperficies ſphæræ circumſcriptæ eſt ad ſuperficiem rhombi quadrati conici, ut ſphæræ circumſcriptæ diameter ad diametrum inſcriptæ, ſeu, ut diameter ſphæræ circumſcriptæ ad latus quadrati rhombum gignentis.

Corollarium VI.

Et, quoniam $A : \mathrm{a} = D : d$, & $a : \mathrm{a} = d : D$, atque convertendo $\mathrm{a} : a = D : d$, erit $A : \mathrm{a} = \mathrm{a} : a$, hoc eſt ſu-

perficies rhombi erit media proportionalis inter ſuperficiem ſphæræ circumſcriptæ, & inſcriptæ.

Corollarium VII.

Quod ſi ſphæra inſcribitur, & circumſcribitur tetrahedro; quia $a = \frac{2D^2}{\sqrt{3}}$, & $d = \frac{D}{3}$, ac proinde $a = \frac{A}{9}$, erit $A : a = \frac{AD^2}{9} : \frac{2D^4}{9\sqrt{3}} = A : \frac{2D^2}{\sqrt{3}}$; ſed, poſita circumferentia circuli maximi ſphæræ circumſcriptæ $= C$, erit $A = CD$; ergo $A : a = CD : \frac{2D^2}{\sqrt{3}} = C : \frac{2D}{\sqrt{3}}$.

Corollarium VIII.

Sed, ſi corpus, cui inſcribitur ſphæra, & circumſcribitur, eſt octahedrum, cum ſit $a = D^2\sqrt{3}$, $d = \frac{D}{\sqrt{3}}$, & $a = \frac{A}{3}$, erit $A : a = \frac{AD^2}{3} : \frac{aD^2}{3} = A : D^2\sqrt{3}$; denominata autem circuli maximi ſphæræ circumſcriptæ circumferentia $= C$, cum ſit $A = CD$, erit $A : a = CD : D^2\sqrt{3} = C : D\sqrt{3}$.

Corollarium IX.

Et quia, inſcripta, circumſcriptaque ſphæra cubo; $a = 2D^2$, $d = \frac{D}{\sqrt{3}}$, & $aD^2 = \frac{AD^2}{3}$, erit $A : a = \frac{AD^2}{3} : \frac{aD^2}{3} = A : 2D^2$; ſed, ſuppoſita circumferentia circuli maximi ſphæræ circumſcriptæ $= C$, cum ſit $A = CD$, erit $A : a = CD : 2D^2 = C : 2D$; & quia, ut $C : 2D$, ita circulus maximus ſphæræ ad quadratum in eo inſcriptum per Corol. II. Theor. II.; ergo ſuperficies ſphæræ circumſcriptæ eſt ad ſuperficiem cubi, ut ejus circulus maximus ad quadratum eidem circulo inſcriptum.

Corollarium X.

Suppoſita ſphæra inſcripta, & circumſcripta icoſahedro, erit $a = \frac{10D^2\sqrt{3}}{5+\sqrt{5}}$, & $d = \frac{3D + D\sqrt{5}}{\sqrt{30+\sqrt{180}}}$, adeoque $aD^2 = A \times \frac{14D^2 + D^2\sqrt{180}}{30+\sqrt{180}}$; ergo $A : a = A \times \frac{14D^2 + D^2\sqrt{180}}{30+\sqrt{180}} : a \times \frac{14D^2 + D^2\sqrt{180}}{30+\sqrt{180}} = A : \frac{10D^2\sqrt{3}}{5+\sqrt{5}}$; ſed, ſi circumferentia circuli maximi ſphæ-

ſphæræ circumſcriptæ dicatur C, erit $A = CD$; ergo $A : a = CD : \frac{10D^2\sqrt{3}}{5+\sqrt{5}} = C : \frac{10D\sqrt{3}}{5+\sqrt{5}}$.

Corollarium XI.

Si tandem ſphæra inſcribitur, & circumſcribitur Dodecahedro, cum ſit $a = \frac{10D^2}{\sqrt{10+2\sqrt{5}}}$, & $d = \frac{3D + D\sqrt{5}}{\sqrt{30+\sqrt{180}}}$, erit $aD^2 = A \times \frac{14D^2 + D^2\sqrt{180}}{30+\sqrt{180}}$; quare $A : a = A \times \frac{14D^2 + D^2\sqrt{180}}{30+\sqrt{180}} : a \times \frac{14D^2 + D^2\sqrt{180}}{30+\sqrt{180}} = A : \frac{10D^2}{\sqrt{10+2\sqrt{5}}}$; quod ſi ſupponatur circumferentia circuli maximi ſphæræ circumſcriptæ $= C$, erit $A = CD$; ergo $A : a = CD : \frac{10D^2}{\sqrt{10+2\sqrt{5}}} = C : \frac{10D}{\sqrt{10+2\sqrt{5}}}$.

Corollarium XII.

Hinc erit ſuperficies tetrahedri ad ſuperficiem octahedri, ut 2 : 3; nam, denominatis corporibus, & ſphæra circumſcripta per eorum litteras initiales, quod præſtabitur etiam in ſequentibus Corollariis, erit

$$T : S = \frac{2D}{\sqrt{3}} : C$$

$$S : O = C : D\sqrt{3};$$

ergo $T : O = \frac{2D}{\sqrt{3}} : D\sqrt{3} = 2D : 3D = 2 : 3.$

Corollarium XIII.

Superficies tetrahedri ad ſuperficiem cubi, ſeu hexahedri, ut $1 : \sqrt{3}$; nam

$$T : S = \frac{2D}{\sqrt{3}} : C$$

$$S : H = C : 2D;$$

ergo $T : H = \frac{2D}{\sqrt{3}} : 2D = 2D : 2D\sqrt{3} = 1 : \sqrt{3}.$

Corollarium XIV.

Superficies tetrahedri ad ſuperficiem icoſahedri, ut $5 + \sqrt{5} : 15$; nam

$$T : S = \frac{2D}{\sqrt{3}} : C$$

$$S : I = C : \frac{10D\sqrt{3}}{5+\sqrt{5}};$$

ergo $T : I = \frac{2D}{\sqrt{3}} : \frac{10D\sqrt{3}}{5+\sqrt{5}} = \frac{1}{\sqrt{3}} : \frac{5\sqrt{3}}{5+\sqrt{5}} = 5+\sqrt{5} : 15.$

Corol-

Corollarium XV.

Tandem erit ſuperficies tetrahedri ad ſuperficiem dodecahedri, ut $1 : \frac{5\sqrt{3}}{\sqrt{10+2\sqrt{5}}}$; nam

$$T : S = \frac{2D}{\sqrt{3}} : C$$

$$S : \Delta = C : \frac{10D}{\sqrt{10+2\sqrt{5}}};$$

ergo $T : \Delta = \frac{2D}{\sqrt{3}} : \frac{10D}{\sqrt{10+2\sqrt{5}}} = \frac{1}{\sqrt{3}} : \frac{5}{\sqrt{10+2\sqrt{5}}} = 1 : \frac{5\sqrt{3}}{\sqrt{10+2\sqrt{5}}}$

Corollarium XVI.

Proportio ſuperficiei octahedri ad ſuperficiem cubi, ſeu hexahedri erit, ut $\sqrt{3} : 2$; nam

$$O : S = D\sqrt{3} : C$$

$$S : H = C : 2D;$$

ergo $O : H = D\sqrt{3} : 2D = \sqrt{3} : 2$.

Corollarium XVII.

Et ſuperficies octahedri erit ad ſuperficiem icoſahedri, ut $5+\sqrt{5} : 10$; quia

O :

$$O : S = D\sqrt{3} : C$$

$$S : I = C : \frac{10D\sqrt{3}}{5+\sqrt{5}};$$

ergo $O : I = D\sqrt{3} : \frac{10D\sqrt{3}}{5+\sqrt{5}} = 1 : \frac{10}{5+\sqrt{5}} = 5+\sqrt{5} : 10$

Corollarium XVIII.

Superficies denique octahedri eſt ad ſuperficiem dodecahedri, ut $\sqrt{3} : \frac{10}{\sqrt{10+2\sqrt{5}}}$; nam

$$O : S = D\sqrt{3} : C$$

$$S : \Delta = C : \frac{10D}{\sqrt{10+2\sqrt{5}}};$$

ergo $O : \Delta = D\sqrt{3} : \frac{10D}{\sqrt{10+2\sqrt{5}}} = \sqrt{3} : \frac{10}{\sqrt{10+2\sqrt{5}}}$

Corollarium XIX.

Ratio ſuperficiei cubi, ſeu hexahedri ad ſuperficiem icoſahedri erit, ut $5+\sqrt{5} : 5\sqrt{3}$; quia

$$H : S = 2D : C$$

$$S : I = C : \frac{10D\sqrt{3}}{5+\sqrt{5}};$$

ergo $H : I = 2D : \frac{10D\sqrt{3}}{5+\sqrt{5}} = 1 : \frac{5\sqrt{3}}{5+\sqrt{5}} = 5+\sqrt{5} : 5\sqrt{3}.$

Corollarium XX.

Et proportio ſuperficiei cubi, ſeu hexahedri ad proportionem ſuperficiei dodecahedri, ut $1 : \frac{5}{\sqrt{10+2\sqrt{5}}}$; nam

$$H : S = 2D : C$$
$$S : \Delta = C : \frac{10D}{\sqrt{10+2\sqrt{5}}};$$
ergo $H : \Delta = 2D : \frac{10D}{\sqrt{10+2\sqrt{5}}} = 1 : \frac{5}{\sqrt{10+2\sqrt{5}}}$.

Corollarium XXI.

Erit tandem ſuperficies icoſahedri, ad ſuperficiem dodecahedri, ut $\frac{\sqrt{3}}{5+\sqrt{5}} : \frac{1}{\sqrt{10+2\sqrt{5}}}$, ſeu, ut $\sqrt{30+6\sqrt{5}} : 5+\sqrt{5}$; nam

$$I : S = \frac{10D\sqrt{3}}{5+\sqrt{5}} : C$$
$$S : \Delta = C : \frac{10D}{\sqrt{10+2\sqrt{5}}};$$
ergo $I : \Delta = \frac{10D\sqrt{3}}{5+\sqrt{5}} : \frac{10D}{\sqrt{10+2\sqrt{5}}} = \frac{\sqrt{3}}{5+\sqrt{5}} : \frac{1}{\sqrt{10+2\sqrt{5}}} = \sqrt{30+6\sqrt{5}} : 5+\sqrt{5}$.

THEO-

THEOREMA IV.

Soliditas corporis, cui inſcribi, & circumſcribi poteſt ſphæra, eſt ad ſoliditatem ſphæræ circumſcriptæ in ratione compoſita rationis ſuperficiei corporis ad ſuperficiem ſphæræ inſcriptæ, & cubi diametri minoris, ad cubum diametri majoris.

Demonſtratio.

Sit ſoliditas ſphæræ circumſcriptæ $= S$
— — — — — inſcriptæ $= s$
— — — — — corporis = *ſ*
diameter ſphæræ circumſcriptæ $= D$
— — — — — inſcriptæ $= d$
ſuperficies ſphæræ circumſcriptæ $= A$
— — — — — inſcriptæ $= a$
— — — — — corporis $= \mathrm{a}$
erit $S : s = D^3 : d^3$;
ſed per Theor. II. Zanotti s : *ſ* $= a : \mathrm{a}$,
& Ss : s*ſ* $= aD^3 : \mathrm{a}d^3$;
ergo S : *ſ* $= aD^3 : \mathrm{a}d^3$. Q. e. d.

Corollarium I.

Si ponatur, corpus, cui inſcribitur, & circumſcribitur ſphæra, eſſe conum æquilaterum, cum ſit $a : \mathrm{a} =$

 4:9,

$4 : 9$, & $d = \frac{D}{2}$, erit $a = \frac{4a}{9}$, & $d^3 = \frac{D^3}{8}$; ergo $S : ſ = \frac{4aD^3}{9} : \frac{aD^3}{8} = \frac{4}{9} : \frac{1}{8} = 32 : 9$.

Corollarium II.

Quod ſi fuerit cylindrus quadratus; quoniam $a = \frac{2a}{3}$, $d = \frac{D}{\sqrt{2}}$, ac proinde $d^3 = \frac{D^3}{\sqrt{8}}$, erit $S : ſ = \frac{2aD^3}{3} : \frac{aD^3}{\sqrt{8}}$; ergo $S : ſ = \frac{2}{3} : \frac{1}{\sqrt{8}} = 2\sqrt{8} : 3 = \sqrt{32} : 3$. Hinc, cum ſoliditas coni æquilateri ſit ad ſoliditatem ſphæræ $= 9 : 32 = \frac{9}{8} : 4$, & ſoliditas ſphæræ ad ſoliditatem cylindri quadrati $= \sqrt{32} : 3 = 4 : \frac{6}{\sqrt{8}}$, erit ſoliditas primi ad ſoliditatem ſecundi $= \frac{9}{8} : \frac{6}{\sqrt{8}}$; ſed $\frac{9}{8} : \frac{6}{\sqrt{8}} = \frac{6}{\sqrt{8}} : 4$; ergo ſoliditas ſphæræ, cylindri quadrati, & coni æquilateri, quæ in ſphæra ſunt inſcripta, hanc continuant proportionem $4 : \frac{6}{\sqrt{8}} : \frac{9}{8}$.

Corollarium III.

Sed, ſi eſt rhombus quadratus conicus; quia $a : a = d : D$, & $d = \frac{D}{\sqrt{2}}$, eſt $S : ſ = dD^3 : Dd^3 = D^2 : d^2 =$

D^2

$D^2 : \frac{D^2}{2} = 2 : 1$; eſt ergo ſoliditas ſphæræ dupla ſoliditate rhombi.

Corollarium IV.

Hinc, cum ſit ſuperficies ſphæræ ad ſuperficiem rhombi inſcripti, ut $\sqrt{2} : 1$ per Corol. V. Theor. III., & ſoliditas unius ad ſoliditatem alterius ut $2 : 1$, ſuperficies ſunt ut radices ſoliditatum.

Corollarium V.

Erunt inſuper ſoliditates ſphæræ circumſcriptæ, rhombi, & coni recti inſcripti in proportione continua, nimirum ut $4 : 2 : 1$.

Corollarium VI.

Inſcripta, circumſcriptaque ſphæra tetrahedro, erit $a = \frac{2D^2}{\sqrt{3}}$, $d = \frac{D}{3}$, ac propterea $a = \frac{A}{9}$; ergo $S : ſ = \frac{AD^3}{9} : \frac{2D^5}{27\sqrt{3}} = A : \frac{2D^2}{3\sqrt{3}}$; ſed, poſita circumferentia circuli maximi ſphæræ circumſcriptæ $= C$, eſt $A = CD$; ergo $S : ſ = CD : \frac{2D^2}{3\sqrt{3}} = C : \frac{2D}{3\sqrt{3}}$.

Corol.

Corollarium VII.

Quare, cum ſuperficies ſphæræ ſit ad ſuperficiem tetrahedri, ut $C : \frac{2D}{\sqrt{3}}$ per Corol. VII. Theor. III., & ſoliditas illius ad ſoliditatem hujus, ut $C : \frac{2D}{3\sqrt{3}}$, erit ratio ſoliditatis ſphæræ ad ſoliditatem tetrahedri inſcripti tripla rationis, quam habet ſuperficies ejuſdem ſphæræ ad ſuperficiem tetrahedri.

Corollarium VIII.

Si octahedro inſcripta, & circumſcripta eſt ſphæra; erit $a = D^2\sqrt{3}$, $d = \frac{D}{\sqrt{3}}$, adeoque $a = \frac{A}{3}$; ergo $S : ſ = \frac{AD^3}{3} : \frac{D^5}{3} = A : D^2$; ſed, ſuppoſita circumferentia circuli maximi ſphæræ circumſcriptæ $= C$, eſt $A = CD$; ergo $S : ſ = CD : D^2 = C : D$.

Corollarium IX.

Et quia ſoliditas rhombi ſolidi eſt dimidium ſoliditatis ſphæræ circumſcriptæ per Corol. III., erit ſoliditas hujus ad ſoliditatem octahedri, ut $\frac{C}{2} : D$, ut $C : 2D$.

Corol-

Corollarium X.

Sed, cum ſit ſuperficies ſphæræ ad ſuperficiem cubi inſcripti, ut $C : 2D$; circulus maximus ſphæræ ad quadratum in eo inſcriptum, ut $C : 2D$ per Corol. IX. Theor. III., erit ſoliditas rhombi ad ſoliditatem octahedri, ut ſuperficies ſphæræ ad ſuperficiem cubi inſcripti, ut circulus maximus ſphæræ ad quadratum inſcriptum.

Corollarium XI.

Inſcripta, circumſcriptaque ſphæra cubo, erit $a = 2D^2$, $d = \frac{D}{\sqrt{3}}$, ac proinde $a = \frac{A}{3}$; ergo $S : ſ = \frac{AD^3}{3} : \frac{2D^5}{3\sqrt{3}} = A : \frac{2D^2}{\sqrt{3}}$; & quoniam, denominata circumferentia circuli maximi C, eſt $A = CD$; ergo $S : ſ = CD : \frac{2D^2}{\sqrt{3}} = C : \frac{2D}{\sqrt{3}}$, ſed etiam ſuperficies ſphæræ eſt ad ſuperficiem tetrahedri, ut $C : \frac{2D}{\sqrt{3}}$ per Corol. VII. Theor. III.; ergo ſuperficies ſphæræ eſt ad ſuperficiem tetrahedri, ut ſoliditas ſphæræ ad ſoliditatem cubi.

Corollarium XII.

Vel icoſahedro; cum ſit $a = \frac{10D^2\sqrt{3}}{5+\sqrt{5}}$, & $d = \frac{3D+D\sqrt{5}}{\sqrt{30+\sqrt{180}}}$, quare $aD^2 = A \times \frac{14D^2+D^2\sqrt{180}}{30+\sqrt{180}}$, erit $S:ſ = AD \times \frac{14D^2+D^2\sqrt{180}}{30+\sqrt{180}} : \frac{10D^2\sqrt{3}}{5+\sqrt{5}} \times \frac{14D^2+D^2\sqrt{180}}{30+\sqrt{180}} \times d = A : \frac{10D\sqrt{3}}{5+\sqrt{5}} \times d$; ſed ſi circumferentia circuli maximi ſphæræ circumſcriptæ dicatur C, eſt $A = CD$; ergo $S:ſ = CD : \frac{10D\sqrt{3}}{5+\sqrt{5}} \times d = C : \frac{10\sqrt{3}}{5+\sqrt{5}} \times d$, & ſubſtituto pro d ejus valore æquali, $d = \frac{3D+D\sqrt{5}}{\sqrt{30+\sqrt{180}}}$; & actu multiplicando, terminoſque ad minimam denominationem reducendo;

$$S:ſ = C : \frac{6D+2D\sqrt{5}}{\sqrt{10+\sqrt{20}}+\sqrt{2+\sqrt{\frac{4}{5}}}}$$

$$= C : \frac{3D+D\sqrt{5}}{\sqrt{\frac{5}{2}+\sqrt{\frac{5}{4}}}+\sqrt{\frac{1}{2}+\sqrt{\frac{1}{20}}}}$$

Corol.

Corollarium XIII.

Vel tandem dodecahedro; quoniam $a = \frac{10D^2}{\sqrt{10+2\sqrt{5}}}$; & $d = \frac{3D+D\sqrt{5}}{\sqrt{30+\sqrt{180}}}$, erit $aD^2 = A \times \frac{14D^2+D^2\sqrt{180}}{30+\sqrt{180}}$; ergo $S : ſ = AD \times \frac{14D^2+D^2\sqrt{180}}{30+\sqrt{180}} : \frac{10D^2}{\sqrt{10+2\sqrt{5}}} \times \frac{14D^2+D^2\sqrt{180}}{30+\sqrt{180}} \times d = A : \frac{10D}{\sqrt{10+2\sqrt{5}}} \times d$; ſed, ſuppoſita circumferentia circuli maximi ſphæræ circumſcriptæ $= C$, eſt $A = CD$; ergo $S : ſ = CD : \frac{10D}{\sqrt{10+2\sqrt{5}}} \times d = C : \frac{10}{\sqrt{10+2\sqrt{5}}} \times d$, & ſubſtituto loco d, ejus valore æquali, $d = \frac{3D+D\sqrt{5}}{\sqrt{30+\sqrt{180}}}$, & actu multiplicando; atque productum ad minores terminos reducendo,

$$S : ſ = C : \frac{3D+D\sqrt{5}}{\sqrt{3+\sqrt{\frac{3}{5}}}}.$$

Corollarium XIV.

Erit ergo ſoliditas tetrahedri ad ſoliditatem octahedri, ut 2 : $\surd 27$; quia

$$T : S = \frac{2D}{3\surd 3} : C$$

$$S : O = C : D;$$

$$\text{ergo } T : O = \frac{2D}{3\surd 3} : D = 2 : 3\surd 3 = 2 : \surd 27.$$

Corollarium XV.

Sed ſuperficies tetrahedri eſt ad ſuperficiem octahedri, ut 2 : 3 per Corol. XII. Theor. III.; ergo ſuperficies illius eſt ad ſuperficiem huius, ut duo ad radicem cubicam viginti ſeptem, & ſoliditas primi ad ſoliditatem ſecundi, ut duo ad radicem quadratam viginti ſeptem.

Corollarium XVI.

Et ſoliditas tetrahedri ad ſoliditatem cubi, ut 1 : 3; nam

$$T : S = \frac{2D}{3\surd 3} : C$$

$$S : H = C : \frac{2D}{\surd 3};$$

$$\text{ergo } T : H = \frac{2D}{3\surd 3} : \frac{2D}{\surd 3} = 1 : 3.$$

Corol-

Corollarium XVII.

Hinc, cum ſit ſuperficies tetrahedri ad ſuperficiem hexahedri, ut $1 : \sqrt{3}$ per Corol. XIII. Theor. III., ſuperficies horum corporum erunt, ut radices ſoliditatum.

Corollarium XVIII.

Soliditas tetrahedri eſt ad ſoliditatem icoſahedri, ut $1 : \frac{9\sqrt{3}+3\sqrt{15}}{\sqrt{10+\sqrt{20}+\sqrt{2+\sqrt{\frac{4}{5}}}}}$; quoniam

$$T : S = \frac{2D}{3\sqrt{3}} : C$$

$$S : I = C : \frac{3D+D\sqrt{5}}{\sqrt{\frac{5}{2}+\sqrt{\frac{5}{4}}}+\sqrt{\frac{1}{2}+\sqrt{\frac{1}{20}}}};$$

ergo $T : I = \frac{2D}{3\sqrt{3}} : \frac{3D+D\sqrt{5}}{\sqrt{\frac{5}{2}+\sqrt{\frac{5}{4}}}+\sqrt{\frac{1}{2}+\sqrt{\frac{1}{20}}}} =$

$$1 : \frac{9\sqrt{3}+\sqrt{15}}{\sqrt{10+\sqrt{20}+\sqrt{2+\sqrt{\frac{4}{5}}}}}.$$

Corollarium XIX.

Soliditas tetrahedri eſt ad ſoliditatem dodecahedri, ut $1 : \frac{9+3\sqrt{5}}{2+\sqrt{\frac{4}{5}}}$; quia

$$T : S = \frac{2D}{3\sqrt{3}} : C$$

$$S : \Delta = C : \frac{3D + D\sqrt{5}}{\sqrt{3} + \sqrt{\frac{3}{5}}};$$

$$\text{ergo } T : \Delta = \frac{2D}{3\sqrt{3}} : \frac{3D + D\sqrt{5}}{\sqrt{3} + \sqrt{\frac{3}{5}}} = 1 : \frac{9 + 3\sqrt{5}}{2 + \sqrt{\frac{4}{5}}}.$$

Corollarium XX.

Soliditas octahedri eſt ad ſoliditatem cubi, ut $\sqrt{3} : 2$; nam

$$O : S = D : C$$

$$S : H = C : \frac{2D}{\sqrt{3}};$$

$$\text{ergo } O : H = D : \frac{2D}{\sqrt{3}} = \sqrt{3} : 2.$$

Corollarium XXI.

Sed ſuperficies octahedri eſt ad ſuperficiem hexahedri, ſeu cubi, ut $\sqrt{3} : 2$ per Corol. XVI. Theor. III.; ergo horum corporum ſuperficies ſunt, ut eorundem ſoliditates.

Corollarium XXII.

Soliditas octahedri eſt ad ſoliditatem icoſahedri, ut $1 : \dfrac{3 + \sqrt{5}}{\sqrt{\frac{5}{2} + \sqrt{\frac{5}{4}}} + \sqrt{\frac{1}{2} + \sqrt{\frac{1}{20}}}}$; quoniam

O:

$$O : S = D : C$$

$$S : I = C : \frac{3D + D\sqrt{5}}{\sqrt{\frac{5}{2} + \sqrt{\frac{5}{4}}} + \sqrt{\frac{1}{2} + \sqrt{\frac{1}{20}}}};$$

ergo $O : I = D : \dfrac{3D + D\sqrt{5}}{\sqrt{\frac{5}{2} + \sqrt{\frac{5}{4}}} + \sqrt{\frac{1}{2} + \sqrt{\frac{1}{20}}}}$

$$= 1 : \frac{3 + \sqrt{5}}{\sqrt{\frac{5}{2} + \sqrt{\frac{5}{4}}} + \sqrt{\frac{1}{2} + \sqrt{\frac{1}{20}}}}.$$

Corollarium XXIII.

Soliditas octahedri eſt ad ſoliditatem dodecahedri; ut $1 : \dfrac{3 + \sqrt{5}}{\sqrt{3} + \sqrt{\frac{3}{5}}}$; nam

$$O : S = D : C$$

$$S : \Delta = C : \frac{3D + D\sqrt{5}}{\sqrt{3} + \sqrt{\frac{3}{5}}};$$

ergo $O : \Delta = D : \dfrac{3D + D\sqrt{5}}{\sqrt{3} + \sqrt{\frac{3}{5}}} = 1 : \dfrac{3 + \sqrt{5}}{\sqrt{3} + \sqrt{\frac{3}{5}}}.$

Corollarium XXIV.

Soliditas cubi eſt ad ſoliditatem icoſahedri, ut $1 : \dfrac{3\sqrt{3} + \sqrt{15}}{\sqrt{10 + \sqrt{20}} + \sqrt{2 + \sqrt{\frac{4}{5}}}}$; quia

$$H : S = \frac{2D}{\sqrt{3}} : C$$

$$S : I = C : \frac{3D + D\sqrt{5}}{\sqrt{\frac{5}{2} + \sqrt{\frac{5}{4}} + \sqrt{\frac{1}{2} + \sqrt{\frac{1}{20}}}}};$$

ergo $H : I = \frac{2D}{\sqrt{3}} : \frac{3D + D\sqrt{5}}{\sqrt{\frac{5}{2} + \sqrt{\frac{5}{4}} + \sqrt{\frac{1}{2} + \sqrt{\frac{1}{20}}}}} = 1 :$

$$\frac{3\sqrt{3} + \sqrt{15}}{\sqrt{10 + \sqrt{20} + \sqrt{2 + \sqrt{\frac{4}{5}}}}}.$$

Corollarium XXV.

Soliditas cubi, ſeu hexahedri eſt ad ſoliditatem dodecahedri, ut $1 : \frac{3 + \sqrt{5}}{2 + \sqrt{\frac{4}{5}}}$; quoniam

$$H : S = \frac{2D}{\sqrt{3}} : C$$

$$S : \Delta = C : \frac{3D + D\sqrt{5}}{\sqrt{3} + \sqrt{\frac{3}{5}}};$$

ergo $H : \Delta = \frac{2D}{\sqrt{3}} : \frac{3D + D\sqrt{5}}{\sqrt{3} + \sqrt{\frac{3}{5}}} = 1 : \frac{3\sqrt{3} + \sqrt{15}}{\sqrt{12} + \sqrt{\frac{12}{5}}}$

$= 1 : \frac{3 + \sqrt{5}}{2 + \sqrt{\frac{4}{5}}}.$

Corol.

Corollarium XXVI.

Soliditas denique icoſahedri eſt ad ſoliditatem dodecahedri, ut 1 : $\dfrac{\sqrt{\frac{5}{2}+\sqrt{\frac{5}{4}}+\sqrt{\frac{1}{2}+\sqrt{\frac{1}{20}}}}}{\sqrt{3}+\sqrt{\frac{3}{5}}}$; nam

$$I : S = \frac{3D+D\sqrt{5}}{\sqrt{\frac{5}{2}+\sqrt{\frac{5}{4}}+\sqrt{\frac{1}{2}+\sqrt{\frac{1}{20}}}}} : C$$

$$S : \Delta = C : \frac{3D+D\sqrt{5}}{\sqrt{3}+\sqrt{\frac{3}{5}}} ;$$

ergo $I:\Delta = \dfrac{3D+D\sqrt{5}}{\sqrt{\frac{5}{2}+\sqrt{\frac{5}{4}}+\sqrt{\frac{1}{2}+\sqrt{\frac{1}{20}}}}} : \dfrac{3D+D\sqrt{5}}{\sqrt{3}+\sqrt{\frac{3}{5}}}$

$$= 1 : \frac{\sqrt{\frac{5}{2}+\sqrt{\frac{5}{4}}+\sqrt{\frac{1}{2}+\sqrt{\frac{1}{20}}}}}{\sqrt{3}+\sqrt{\frac{3}{5}}} .$$

THEOREMA V.

Tota ſuperficies cujuslibet priſmatis cylindro circumſcripti eſt ad ſuperficiem integram cylindri, ut perimeter baſis priſmatis ad circumferentiam baſis cylindri.

Demon-

Demonſtratio .

Superficies priſmatis, & cylindri duo adæquant rectangula, quorum baſes ſunt perimetri baſium utriuſque corporis, & altitudo eſt communis, ſecluſis tamen baſibus; ſed hæc rectangula, cum ſint æque alta, ſunt, ut baſes; ergo ſuperficies priſmatis eſt ad ſuperficiem cylindri ſine baſibus, ut perimeter baſis priſmatis ad circumferentiam baſis cylindri; ſed per Theor. I. Zanotti baſes etiam ſunt in eadem perimetrorum ratione; ergo, tam priſmati, quam cylindro addita baſi, erit integra ſuperficies priſmatis ad integram cylindri, ut perimeter baſis priſmatis ad peripheriam baſis cylindri. Q. e. d.

Corollarium I.

Sed ſoliditas priſmatis eſt ad cylindri æque alti ſoliditatem, ut baſis priſmatis ad baſim cylindri; ergo horum corporum ſoliditates, ſuperficies, baſes, & baſium ambitus ſunt in eadem proportione.

Corollarium II.

Si priſma cylindro circumſcriptum triangulum æquilaterum habebit pro baſi, & priſmati alius erit circumſcri-

ſcriptus cylindrus, ſuperficies cylindri circumſcripti ad ſuperficiem priſmatis inſcripti, non conſideratis baſibus, duplam habebit rationem ea, quam habet ſuperficies integra cylindri inſcripti ad ſuperficiem integram priſmatis per hoc Theorema, & per Corol. II., & III. Theor. I.

THEOREMA VI.

Tota ſuperficies pyramidis cono circumſcriptæ eſt ad ſuperficiem integram coni, ut perimeter baſis pyramidis ad circumferentiam baſis coni.

Demonſtratio.

Superficies pyramidis, & coni ſine baſibus ſunt æquales duobus triangulis, quorum altitudo eſt communis, & baſes ſunt ut perimetri baſium; ſed hæc duo triangula æque alta ſunt ut baſes; ergo ſuperficies pyramidis eſt ad coni ſuperficiem, excluſis baſibus, ut perimeter baſis unius ad circumferentiam baſis alterius; & additis baſibus, quæ ſunt in eadem perimetrorum proportione per Theor. I. Zanotti, tota ſuperficies pyramidis eſt ad coni ſuperficiem, ut ambitus baſis pyramidis ad peripheriam baſis coni. Q. e. d.

Corollarium I.

Et, quia ambo corpora pyramis, & conus ſunt æque alta, eorum ſoliditates ſunt, ut baſes; ergo ſoliditates ſunt, ut ſuperficies, atque ſoliditates, ſuperficies, baſes, & perimetri baſium eandem habent rationem.

Corollarium II.

Satis manifeſtum eſt illud idem, quod dictum fuit in Corol. II. Theor. præc., verum eſſe etiam de ſuperficie coni circumſcripti reſpectu ad ſuperficiem pyramidis, cujus baſis eſt triangularis, & æquilatera.

THEOREMATA
VARIA

Ad Rectangulorum Summas, ac Differentias in Circulo pertinentia.

UTiliſſima ſemper inter reliquas Geometriæ propoſitiones ea judicarunt Theoremata Mathematici, quæ de linearum potentiis, hoc eſt de quadratis agunt, quæque rectangula varia e rectarum aut bifariam, aut utcumque diviſarum partibus naſcentia conferunt cum rectangulis integrarum linearum, vel quadratis, vel alterius lineæ ſegmentis. Hinc celeberrimus apud ipſos eſt totus Euclidis liber ſecundus, qui hæc continet; propoſitiones item trigeſima quinta, & trigeſima ſexta libri tertii, aliæque huiuſmodi; illumque præcipuarum operationum algebraicarum fundamentum appellare, atque uberrimum veritatum calculo detegendarum fontem prædicare conſueverunt. En igitur tibi in hoc Opuſculo tredecim Theoremata, quæ de rebus ab his non alienis pertractant. In prioribus ſiquidem, ductis in circulo, determinata quadam ratione, lineis vel numero finitis, vel infinitis,

modo rectangulorum ex earum ſegmentis productorum differentias inter ſe invicem, ſeu a rectangulo omnium maximo, modo eorum ſummas conſidero; aliquando ſummas linearum, earumque differentias vel inter ſe, vel a diametro examini ſubiicio. In reliquis vero circulum cum aliqua ex Sectionibus conicis, vel cum Cycloide ita comparo, ut diameter circuli, & axis curvæ eadem ſit linea; ductiſque tum circuli, tum alterius curvæ ordinatis, rectangulorum proprietates ex ſumma ſemiordinatarum utriuſque curvæ in earundem differentiam trado. Theorematibus autem omnibus geometrica demonſtratione firmatis, plura alia in eorum Corollariis, ac Scholiis deduco; & potiſſimum ex primis nonnulla ad accelerationem motus gravium pertinentia; aliqua inſuper de curva, quam deſcribunt gravia horizontaliter proiecta. Nec difficile eſſet ex iiſdem principiis aliquam inferre propoſitionem, quæ in Mechanica traditur, de viribus centrifugis, ac centripetis. Ex alteris vero, ſuppoſitis circulo, & alia curva circa proprium axem revolutis, plures corporum, quæ inde oriuntur, proportiones aſſigno. Fateor equidem, plurimas ex his, quas ſcribo, veritates faciliori ratione erui poſſe ope calculi integralis; puto tamen, hunc geometricæ rerum demonſtrandarum rationi non eſſe præferendum; cum per calculum quælibet inventa veritas ſe ſe exhibeat ſine ullo ut plurimum ad alias veritates reſpectu: non ita

vero,

vero, dum veritas aliqua ex geometricis principiis eliciitur; Tunc enim ſponte ſua innumeræ ſe ſe nobis offerunt relationes, quarum uſu mathematica facultas mirificè amplificatur.

THEOREMA I.

Sit in circulo AOCZ (Fig. 1. 2. 3.) chorda ZO, quæ ſecta bifariam ſit in B, ducta ABC, & huic parallelis, vel in unum punctum concurrentibus, vel ſe mutuo ſecantibus DEF, GHI, LMN, erunt differentiæ rectangulorum DEF, GHI, LMN a rectangulo ABC æquales quadratis diſtantiarum BE, BH, BM ab eodem rectangulo.

Demonſtratio.

$AB \times BC = ZB \times BO = BO^2$, ob $BZ = BO$ per hyp., ſed $DE \times EF = ZE \times EO$, & $ZE \times EO + BE^2 = BO^2 = AB \times BC$; ergo $AB \times BC - DE \times EF = BE^2$. Pariter $GH \times HI = ZH \times HO$, & $ZH \times HO + BH^2 = BO^2 = AB \times BC$; ergo $AB \times BC - GH \times HI = BH^2$. Cum igitur idem conſtanter demonſtretur de omnibus rectangulis ſegmentorum per ſemichordam BO, conſtat differentias horum rectangulorum a rectangulo maximo æquales eſſe quadratis diſtantiarum ab eodem rectangulo. Q. e. d.

Corol-

Corollarium I.

Rectangulum ſegmentorum ad punctum O ſemper erit æquale zero: pariter differentia rectanguli ABC a ſe ipſo ſemper erit æqualis zero. Ergo ſumma differentiarum rectangulorum ABC, DEF, GHI, LMN, o a rectangulo ABC $= o + BE^2 + BH^2 + BM^2 + BO^2$.

Corollarium II.

Poteſt ſemichorda BO ſupponi diviſa in partes infinite parvas, & æquales; ergo ſumma differentiarum infinitorum rectangulorum ABC, DEF, &c. in infin. a rectangulo ABC æqualis erit ſummæ quadratorum $o + BE^2 + BH^2$ &c. in infinitum, quorum maximum erit BO^2; ſed ratio, quam habet ſumma omnium quadratorum ab o in infinitum continuatorum ad ſummam totidem maximo æqualium eſt ut 1 : 3; ergo ſumma infinitarum differentiarum dictorum rectangulorum ad ſummam quadratorum totidem maximo BO^2 æqualium eſt in aſſignata ratione.

Corollarium III.

Summa omnium rectangulorum ABC, DEF, GHI &c., quorum ultimum eſt zero, æqualis eſt rectangulo

ABC,

ABC, ſeu BO^2 toties accepto, quotus eſt numerus dictorum rectangulorum, demptis quadratis diſtantiarum ab eodem rectangulo ABC; nam ABC + o = ABC, DEF + BE^2 = ABC, GHI + BH^2 = ABC, LMN + BM^2 = ABC, o + BO^2 = ABC; ergo ſumma = 5ABC — o — BE^2 — BH^2 — BM^2 — BO^2.

Corollarium IV.

Suppoſita ſemichorda BO diviſa in partes infinite parvas, & æquales, ſumma infinitorum rectangulorum ABC, DEF &c. in infin. æqualis erit rectangulo ABC, ſeu BO^2 toties accepto, quot ſunt quadrata ab o in infinitum continuata, ſubtractis iiſdem infinitis quadratis; ſed ſumma quadratorum ab o in infinitum continuatorum eſt tertia pars ſummæ quadratorum totidem maximo æqualium; ergo ſumma horum rectangulorum eſt ad ſummam totidem maximo æqualium, ut 2 : 3; & eorundem differentia ad ipſorum ſummam, ut 1 : 2.

Corollarium V.

Sit BO ſemichorda, & Ia (Fig. 4.) altera ſemichorda; utraque ſupponatur diviſa in partes infinite parvas numero æquales; erunt ſummæ rectangulorum ex ſegmentis per utramque ſemichordam, & differentiarum

a ma-

a maximo inter ſe, ut cubi ſemichordarum; nam, claritatis gratia, ſumma quadratorum ab o in infinitum continuatorum $o + BE^2 + BM^2$ &c. in infin., cum maximus terminus ſit BO, eſt æqualis $\frac{BO^3}{3} + \frac{BO^2}{2} + \frac{BO}{6}$; & ſumma quadratorum ab o in infinitum continuatorum $o + IS^2 + IY^2$ &c. in infinitum, cum maximus terminus ſit Ia, æqualis eſt $\frac{Ia^3}{3} + \frac{Ia^2}{2} + \frac{Ia}{6}$; ſed termini $\frac{BO^3}{3}$, & $\frac{Ia^3}{3}$ ſunt infinite majores, quam $\frac{BO^2}{2}$, & $\frac{BO}{6}$, $\frac{Ia^2}{2}$, & $\frac{Ia}{6}$, adeoque pro nihilo habendi; ergo ſumma prima eſt ad ſummam ſecundam = $\frac{BO^3}{6} : \frac{Ia^3}{3}$; ſed hæ ex dictis ſunt ſummæ differentiarum rectangulorum a maximo; ergo huiuſmodi ſummæ differentiarum ſunt, ut $\frac{BO^3}{3} : \frac{Ia^3}{3}$, ſeu $BO^3 : Ia^3$; ſed ex dictis pariter ſummæ differentiarum ſunt ad ſummam rectangulorum, ut $1 : 2 = \frac{BO^3}{3} : \frac{2BO^3}{3} = \frac{Ia^3}{3} : \frac{2Ia^3}{3}$; ergo etiam ſumma rectangulorum ABC, DEF, LMN &c. in infinitum eſt ad ſummam rectangulorum HIK, PSG, VYd &c. in infinitum, ut $BO^3 : Ia^3$.

Corol-

Corollarium VI.

Si ad punctum A ducitur tangens SA, (Fig. 5.) & producuntur lineæ OT in S, NL in R &c., eſt rectangulum OST $= SA^2$, NRL $= RA^2$ &c.; ergo ſumma rectangulorum OST, NRL &c. æqualis eſt ſummæ quadratorum $SA^2 + RA^2$ &c., quorum ultimum in A eſt zero; ſed ſumma rectangulorum $o + LMN + GHI + DEF + ABC = 5BO^2 - o - BE^2 - BH^2 - BM^2 - BO^2$; ergo ſumma illorum rectangulorum eſt ad ſummam iſtorum, ut ſumma quadratorum $SA^2 + RA^2 + QA^2 + PA^2 + o : 5BO^2 - o - BE^2 - BH^2 - BM^2 - BO^2$.

Corollarium VII.

Quod ſi tum tangens, tum ſemichorda BO ſit diviſa in partes infinite parvas, & æquales, erit ſumma rectangulorum OST, NRL &c. in infin. $= \frac{SA^3}{3}$, ſumma rectangulorum ABC, DEF &c. in infin. $= \frac{2BO^3}{3}$; ergo illa erit ad hanc, ut $SA^3 : 2BO^3$; & illa eadem ad ſummam differentiarum ABC, DEF &c. in infinitum a rectangulo maximo ABC, ut $SA^3 : BO^3$.

Corollarium VIII.

Si ABC, & ZBO (Fig. 6.) ſunt duæ diametri ſe ſecantes ad angulos rectos, ſeu AIC ſecat chordam RIP bifariam, & ad angulos rectos, & ſupponuntur tum tangens, tum ſemichorda, ſeu ſemidiameter diviſæ in partes infinite parvas, & æquales, reliquæ vero rectæ DEp, LNb &c. parallellæ, tunc erit $YA = BO$, $FA = RI$, quare erit ſumma omnium rectangulorum $YO^2 + RF\omega$ &c. in infinitum $= \frac{YA^3}{3}$; & ſumma omnium rectangulorum ABC, HEp &c. in infinitum $= \frac{2BO^3}{3} = \frac{2YA^3}{3}$; ergo illa erit ad hanc, ut $\frac{YA^3}{3} : \frac{2YA^3}{3} = 1 : 2$, ut quadratum inſcriptum ad circumſcriptum. Idem demonſtratur, & eodem modo de ſumma rectangulorum RFω &c. reſpectu ſummę rectangulorum AIC, HKp &c.

THEOREMA II.

Iiſdem ſuppoſitis, quæ in Theoremate præcedenti, differentiæ rectangulorum inter ſe invicem ſunt æquales differentiis quadratorum diſtantiarum a maximo, hoc eſt differentia inter rectangulum ABC, & DEF eſt $BE^2 - 0$, inter DEF, & GHI eſt $BH^2 - BE^2$. Fig. 1. 2. 3.

Demon-

Demonſtratio.

Hoc Theorema eſſe poſſet Corollarium præcedentis, ſed, ut faciliori ratione procedam, idem ita demonſtro: $ABC + o = ABC$, $DEF + BE^2 = ABC$, $GHI + BH^2 = ABC$, $LMN + BM^2 = ABC$, $o + BO^2 = ABC$; ergo $ABC + o = DEF + BE^2 = GHI + BH^2 = LMN + BM^2 = o + BO^2$; ergo $ABC - DEF = BE^2 - o$, $DEF - GHI = BH^2 - BE^2$, $GHI - LMN = BM^2 - BH^2$, $LMN - o = BO^2 - BM^2$. Q. e. d.

Corollarium I.

Erit ergo ſumma differentiarum inter rectangula ABC, & DEF, DEF, & GHI &c. æqualis rectangulo maximo ABC, ſeu quadrato ſemichordæ BO; idque, ſive ſemichorda ſit diviſa in partes finitas, ſive infinitas, ac proinde ſive numero finita ſint rectangula, ſive infinita; nam ſumma omnium differentiarum $= BE^2 - o + BH^2 - BE^2 + BM^2 - BH^2 + BO^2 - BM^2$; ergo æqualis BO^2, ſeu ABC.

Corollarium II.

Si ſemichorda BO eſt diviſa in partes numero finitas, vel infinitas, ſed æquales, differentiæ rectangulorum

rum inter ſe invicem conſtituunt progreſſionem arithmeticam numerorum imparium; differentiæ enim quadratorum numerorum naturalium ſunt numeri impares.

Corollarium III.

Summa differentiarum rectangulorum ABC, DEF &c. a maximo ABC eſt ad ſummam differentiarum eorundem inter ſe, ut ſumma quadratorum diſtantiarum a maximo ad ipſorum differentiam.

Corollarium IV.

Si ABC, & ZBO (Fig. 7.) ſunt circuli diametri, ſumma differentiarum rectangulorum ABC, HEP &c. in ſemicirculo æqualis eſt quadrato radii, in toto circulo duplo quadrato radii; ſed duplum quadratum radii æquale eſt quadrato circulo inſcripto; ergo ſumma differentiarum rectangulorum ABC, HEP, hep in toto circulo æqualis eſt quadrato inſcripto: eſt autem quadratum inſcriptum ad circulum, ut diameter ad ſemicircumferentiam; ergo ſumma differentiarum omnium rectangulorum ex ſegmentis per diametrum eſt ad circulum, ut diameter ad ſemicircumferentiam.

Corol-

Corollarium V.

Ex dictis conſtat, quod ſi in eodem, vel diverſis circulis ducuntur plures chordæ, quæ utcumque dividantur, & per puncta diviſionum ducuntur lineæ utrimque ad circumferentiam terminatæ, ſummæ differentiarum rectangulorum ex ſegmentis inter ſe invicem ſunt, ut quadrata ſubtenſarum; quod ſi chordæ in diverſis circulis ſecabunt arcus ſimiles, eædem erunt, ut quadrata radiorum, hoc eſt, ut ipſi circuli.

Corollarium VI.

Si duo circuli GMHN, LMEN (Fig. 8.) ſe mutuo ſecant, ducta ſubtenſa communi MN, & per eius puncta rectis EG, RS, &c. ad circumferentiam concavam utriuſque circuli terminantibus, cum rectangulum EIL ſit æquale GIH, ROT = SOQ, atque ita conſtanter; erit ſumma rectangulorum, item ſumma differentiarum a maximo rectangulo, & inter ſe in uno circulo, æqualis ſummæ rectangulorum, item differentiarum a maximo, & inter ſe in altero circulo.

THEOREMA III.

Si duo circuli GMHN, EMLN ſe mutuo interſecant, ducta a puncto M ad N ſubtenſa MN, & alia qua-

quacumque recta eam ſecante ROS, erit RQ : ST = QO : OT.

Demonſtratio.

RO×OT = MO×NO, QO×SO = MO×NO; ergo RO×OT = QO×SO, & RO : QO = SO : OT; ſed dividendo RO — QO : QO = SO — OT : OT; ergo RQ : QO = ST : OT; & alternando RQ : ST = QO : OT. Q. e. d.

Corollarium I.

Patet eſſe RO : SO = RQ : ST; nam RO : QO = SO : OT; & alternando RO : SO = QO : OT; ſed etiam RQ : ST = QO : OT; ergo RO : SO = RQ : ST.

Corollarium II.

Hinc data qualibet linea EG utcumque ſecta in H, & L, ſi EH ſit minor EL, ſecari taliter poterit in I, ut ſit EH : GL = HI : LI, ſeu EH : GL = EI : GI, hoc modo: Bifariam dividatur EL in C. Centro C, radio CL deſcribatur circulus EMLN: rurſus bifariam dividatur GH in F, & centro F, radio FG deſcribatur circulus GMHN priorem ſecans in M, & N; a puncto M ad N ducatur MN dico factum. Patet ex theoremate.

THEO-

THEOREMA IV.

Dato circulo DFIR (Fig. 9.) ſi in eo ducitur diameter RD, atque ab eius extremo R ad quæcumque circumferentiæ puncta rectæ RI, RF, RG, RH, dico, ducta tangente RX, & ex punctis H, G, F, I, parallelis HS, GT, FV, IX uſque ad tangentem RX productis, eſſe $IX : FV = RI^2 : RF^2$; $FV : GT = RF^2 : RG^2$; $GT : HS = RG^2 : RH^2$; $HS : DR = RH^2 : RD^2$.

Demonſtratio.

Ducantur lineæ DI, DF, DG, DH, erit triangulum XIR ſimile IRD, FVR ſimile FRD, GTR ſimile RGD, HSR ſimile RHD; ergo $IX : IR = IR : RD$; $FV : FR = FR : RD$; $GT : GR = GR : RD$; $HS : HR = HR : RD$; ergo $IX \times RD = IR^2$, $FV \times RD = FR^2$, $GT \times RD = GR^2$, $HS \times RD = HR^2$; ergo $IX \times RD : FV \times RD = IR^2 : FR^2$; $GT \times RD : HS \times RD = GR^2 : HR^2$; ſed $IX \times RD : FV \times RD = IX : FV$, $GT \times RD : HS \times RD = GT : HS$; ergo $IX : FV = IR^2 : FR^2$, $FV : GT = FR^2 : GR^2$, $GT : HS = GR^2 : HR^2$, $HS + RD = HR^2 : DR^2$. Q. e. d.

Corol-

Corollarium I.

Erit ergo ſumma linearum $IX + FV + GT + HS + DR : DR = IR^2 + FR^2 + GR^2 + HR^2 + DR^2 : DR^2$; nam quia $IX : FV = IR^2 : FR^2$, erit componendo $IX + FV : FV = IR^2 + FR^2 : FR^2$; ſed $FV : GT = FR^2 : GR^2$; ergo ex æquo $IX + FV : GT = IR^2 + FR^2 : GR^2$; & iterum componendo $IX + FV + GT : GT = IR^2 + FR^2 + GR^2 : GR^2$; ſed $GT : HS = GR^2 : HR^2$; ergo ordinate $IX + FV + GT : HS = IR^2 + FR^2 + GR^2 : HR^2$; & denuo componendo $IX + FV + GT + HS : HS = IR^2 + FR^2 + GR^2 + HR^2 : HR^2$; ſed $HS : DR = RH^2 : RD^2$; ergo $IX + FV + GT + HS : DR = IR^2 + FR^2 + GR^2 + HR^2 : DR^2$; & tandem componendo $IX + FV + GT + HS + DR : DR = IR^2 + FR^2 + GR^2 + HR^2 + DR^2 : DR^2$.

Corollarium II.

Ex Corollario præcedente conſtat eſſe $IX + FV + GT + HS + DR : DR = IR^2 + FR^2 + GR^2 + HR^2 + DR^2 : DR^2$; ergo

$$IX + FV + GT + HS + DR = \frac{IR^2 + FR^2 + GR^2 + HR^2 + DR^2}{DR^2} \times DR = \frac{IR^2 + FR^2 + GR^2 + HR^2 + DR^2.}{DR}$$

Corol-

Corollarium III.

Eſt autem $IR^2 = DR^2 - DI^2$, $FR^2 = DR^2 - DF^2$, $GR^2 = DR^2 - DG^2$, $HR^2 = DR^2 - DH^2$; $DR^2 = DR^2 - 0$; ergo $IX + FV + GT + HS + DR = \frac{5DR^2 - DI^2 - DF^2 - DG^2 - DH^2 - 0}{DR} = 5DR - \frac{DI^2 - DF^2 - DG^2 - DH^2 - 0}{DR}$: Eſt igitur ſumma omnium parallelarum a quibuſcumque ſemicircumferentiæ punctis ad tangentem ductarum, incluſa diametro, æqualis diametro totidem acceptæ, quot ſunt quadrata ex chordis ab extremo diametri ad dictas parallelas ductis, quarum minima ſemper erit zero, dempto quoto, qui oritur ex diviſione eorundem quadratorum chordarum per diametrum. Determinantur hæ chordæ a punctis in ſemicircumferentia acceptis, & extremo diametri, per quod non eſt ducta tangens.

Corollarium IV.

Si chordæ DH, DG, DF, DI, conſtituunt ſeriem numerorum naturalem 1. 2. 3. 4., erit ſumma linearum DR, HS, GT &c. $= 5DR - \frac{DI^3}{3DR} - \frac{DI^2}{2DR} - \frac{DI}{6DR}$;

H nam

nam per Corol. præced. $IX + FV + GT$ &c. $= 5DR - \frac{DI^2 - DF^2 - DG^2 - DH^2 - o}{DR}$; ſed $DI^2 + DF^2 + DG^2 + DH^2 + o = \frac{DI^3}{3} + \frac{DI^2}{2} + \frac{DI}{6}$; ergo $\frac{DI^2 + DF^2 + DG^2 + DH^2 + o}{DR} = \frac{DI^3}{3DR} + \frac{DI^2}{2DR} + \frac{DI}{6DR}$; ac proinde $IX + FV + GT$ &c. $= 5DR - \frac{DI^3}{3DR} - \frac{DI^2}{2DR} - \frac{DI}{6DR}$.

Corollarium V.

Suppoſita qualibet chorda DI diviſa in partes infinite parvas, & æquales, ſi fient chordæ DH, DG, DF æquales uni, duabus, tribus partibus infiniteſimis ipſius DI &c. in infinitum, erit ſumma omnium linearum IX, FV &c. æqualis tot diametris DR, quot ſunt quadrata chordarum, demptis infinitis quadratis earundem chordarum per diametrum diviſis; ſed ſumma infinitorum quadratorum ab zero æqualis tertiæ parti cubi maximi termini, æqualis $\frac{DI^3}{3}$; ergo, cum numerus quadratorum chordarum æqualis DI, erit ſumma omnium linearum $= DI \times DR - \frac{DI^3}{3DR}$.

Corol-

Corollarium VI.

Quælibet circuli chorda eſt diametri ſubmultiplex; poſita ergo diametro DR $= m$, & qualibet chorda DI $= \frac{m}{r}$, & ſuppoſita eadem chorda DI diviſa in partes infinite parvas, & æquales, erit ſumma omnium linearum IX + FV + GT &c. æqualis $\frac{m^2}{r} - \frac{m^2}{3r^3}$.

Corollarium VII.

Si ſemicircumferentia circuli DFIR (Fig. 16.) ſupponitur diviſa in partes infinite parvas, & æquales, & a ſingulis partibus ducuntur parallelæ diametro uſque ad tangentem productæ, ſumma omnium linearum æqualis eſt producto ex ſemicircumferentia in diametrum, dempto quoto ex diviſione orto quadratorum chordarum per diametrum arcubus unius, duarum, trium partium infinitarum in ſemicircumferentia correſpondentium. Et generatim quælibet ſumma linearum a ſingulis partibus infinite parvis, & æqualibus alicuius arcus ad tangentem ductarum æqualis eſt producto ex diametro in dictum arcum, ſubtracto eodem quoto ex diviſione per diametrum quadratorum chordarum arcuum unius, duarum, trium partium infinitarum. Sic, ſi DFI

eſt quarta circumferentiæ pars, ſumma linearum ex ſingulis partibus dicti arcus infinite parvis ad tangentem RX ductarum æqualis eſt $DFI \times DR - \overline{DI^2 - DF^2 - DG^2}$ &c. in infinitum diviſis per DR.

THEOREMA V.

Dato circulo DFIR (Fig. 11.) ductiſque lineis DR, HS, GT, FX, ut in ſuperiori Theoremate, erunt differentiæ harum linearum inter ſe, ut differentiæ quadratorum DR^2, HR^2 &c., hoc eſt $DR - HS : HS - GT = DR^2 - HR^2 : HR^2 - GR^2$, $HS - GT : GT - FX = HR^2 - GR^2 : GR^2 - FR^2$.

Demonſtratio.

Cum per ſuperius Theorema ſit $DR : HS = DR^2 : HR^2$, $HS : GT = HR^2 : GR^2$, $GT : FX = GR^2 : FR^2$, erit dividendo $DR - HS : HS = DR^2 - HR^2 : HR^2$; ſed $HS : GT = HR^2 : GR^2$, & per converſionem rationis $HS : HS - GT = HR^2 : HR^2 - GR^2$ ergo erit ex æquo $DR - HS : HS - GT = DR^2 - HR^2 : HR^2 - GR^2$. Pariter $HS - GT : GT = HR^2 - GR^2 : GR^2$; ſed $GT : FX = GR^2 : FR^2$, & per converſionem rationis $GT : GT - FX = GR^2 : GR^2 - FR^2$; ergo ex æquo $HS - GT : GT - FX =$

HR²

$HR^2 - GR^2 : GR^2 - FR^2$; ergo $DR - HS : HS - GT = DR^2 - HR^2 : HR^2 - GR^2$, $HS - GT : GT - FX = HR^2 - GR^2 : GR^2 - FR^2$. Q. e. d.

Corollarium I.

Cum ſit $DR - HS : HS - GT = DR^2 - HR^2 : HR^2 - GR^2$, erit componendo $DR - HS + HS - GT : HS - GT = DR^2 - HR^2 + HR^2 - GR^2 : HR^2 - GR^2$; ſed $HS - GT : GT - FX = HR^2 - GR^2 : GR^2 - FR^2$; ergo $DR - HS + HS - GT : GT - FX = DR^2 - HR^2 + HR^2 - GR^2 : GR^2 - FR^2$: & iterum componendo $DR - HS + HS - GT + GT - FX : GT - FX = DR^2 - HR^2 + HR^2 - GR^2 + GR^2 - FR^2 : GR^2 - FR^2$; ergo $DR - FX : GT - FX = DR^2 - FR^2 : GR^2 - FR^2$; ſed $DR^2 - FR^2 = DF^2$, $GR^2 - FR^2 = DF^2 - DG^2$; ergo $DR - FX : GT - FX = DF^2 : DF^2 - DG^2$; hoc eſt ſumma omnium differentiarum linearum DR, HS &c. eſt ad differentiam penultimæ GT ad ultimam FX, ut quadratum majoris chordæ DF ad differentiam inter quadratum hujus, & chordæ proxime minoris.

Corol-

Corollarium II.

Me non monente, patet ſummam differentiarum ſemper æqualem eſſe lineæ AF interceptæ inter chordam majorem DF, & tangentem AD; eſt enim æqualis DR — FX: patet etiam ſummam differentiarum ſemper æqualem eſſe AF, ſive finitæ, ſive infinitæ ſint lineæ diametro parallelæ ductæ inter D, & F.

Corollarium III.

Cum exceſſus maximæ DR ſupra minimam FX ſit æqualis AF, erit differentia maximæ a minima æqualis ſummæ differentiarum inter DR, & HS, HS, & GT &c.

Corollarium IV.

Per hoc Theorema eſt $DR - HS : HS - GT = DR^2 - HR^2 : HR^2 - GR^2$, $HS - GT : GT - FX = HR^2 - GR^2 : GR^2 - FR^2$. Eſt autem $DR^2 - HR^2 = DH^2$, $HR^2 - GR^2 = DG^2 - DH^2$, $GR^2 - FR^2 = DF^2 - DG^2$; ergo ſi chordæ DH, DG, DF ſecundum ſeriem numerorum naturalem procedunt, differentiæ linearum DR, HS, GT &c. erunt inter ſe invicem, ut numeri impares.

THEO-

THEOREMA VI.

Dato circulo DFIR, ductiſque lineis DR, HS, GT, FX, ut in ſuperiori Theoremate, differentiæ harum linearum a diametro DR ſunt, ut differentiæ quadratorum DR^2, HR^2, GR^2 &c., hoc eſt, ut $DR^2 - HR^2$ ad $DR^2 - GR^2$, ſeu, ut DH^2 ad DG^2 &c.

Demonſtratio.

Eſt $DR : HS = DR^2 : HR^2$, $HS : GT = HR^2 : GR^2$, $GT : FX = GR^2 : FR^2$; ergo $DR : GT = DR^2 : GR^2$, & $DR : FX = DR^2 : FR^2$, atque dividendo, ac invertendo utramque hanc proportionalitatem eſt $GT : DR - GT = GR^2 : DR^2 - GR^2$, & $FX : DR - FX = FR^2 : DR^2 - FR^2$; ſed eſt $DR - HS : HS = DR^2 - HR^2 : HR^2$, $HS : GT = HR^2 : GR^2$, $GT : DR - GT = GR^2 : DR^2 - GR^2$; ergo $DR - HS : DR - GT = DR^2 - HR^2 : DR^2 - GR^2$; ſed inſuper eſt $DR - GT : GT = DR^2 - GR^2 : GR^2$, $GT : FX = GR^2 : FR^2$, $FX : DR - FX = FR^2 : DR^2 - FR^2$; ergo $DR - GT : DR - FX = DR^2 - GR^2 : DR^2 - FR^2$. Eſt autem $DR^2 - HR^2 = DH^2$, $DR^2 - GR^2 = DG^2$, $DR^2 - FR^2 = DF^2$; ergo $DR - HS : DR - GT = DH^2 : DG^2$, & $DR - GT : DR - FX = DG^2 : DF^2$. Q. e. d.

Corol.

Corollarium I.

Summa ergo omnium differentiarum DR, HS, GT, FX a diametro DR eſt ad differentiam maximæ DR a minima FX, ut ſumma quadratorum chordarum $o + DH + DG + DF$ ad quadratum chordæ majoris DF; nam $DR - DR : DR - HS = o : DH^2$, & componendo $DR - DR + DR - HS : DR - HS = o + DH^2 : DH^2$; ſed $DR - HS : DR - GT = DH^2 : DG^2$; ergo $DR - DR + DR - HS : DR - GT = o + DH^2 : DG^2$, & componendo $DR - DR + DR - HS + DR - GT : DR - GT = o + DH^2 + DG^2 : DG^2$; ſed iterum $DR - GT : DR - FX = DG^2 : DF^2$; ergo $DR - DR + DR - HS + DR - GT : DR - FX = o + DH^2 + DG^2 : DF^2$; & rurſus componendo $DR - DR + DR - HS + DR - GT + DR - FX : DR - FX = o + DH^2 + DG^2 + DF^2 : DF^2$, hoc eſt $4DR - DR - HS - GT - FX : DR - FX = o + DH^2 + DG^2 + DF^2 : DF^2$.

Corollarium II.

$DR - FX$ eſt ſumma differentiarum linearum DR, HS &c. inter ſe invicem; ergo ſumma differentiarum linearum DR, HS &c. a diametro DR eſt ad ſummam dif-

differentiarum earundem inter ſe, ut ſumma quadratorum chordarum ad quadratum chordæ majoris.

Corollarium III.

Summa differentiarum linearum DR, HS, GT, FX a diametro DR æqualis eſt ſummæ quadratorum chordarum per diametrum diviſæ; nam ſumma differentiarum linearum DR, HS, GT &c. æqualis eſt 4DR — DR — HS — GT — FX; ſed DR + HS + GT + FX $= \frac{4DR - DF^2 - DG^2 - DH^2 - o}{DR}$; ergo ſumma differentiarum DR + HS &c. a diametro DR $= 4DR - \frac{4DR + DF^2 + DG^2 + DH^2 + o}{DR} = \frac{DF^2 + DG^2 + DH^2 + o}{DR}$.

Corollarium IV.

Quod ſi chordæ DF, DG, DH &c. conſtituunt progreſſionem arithmeticam naturalem, ſumma differentiarum ſupradictarum æqualis erit $\frac{DF^3}{3DR} + \frac{DF^2}{2DR} + \frac{DF}{6DR}$; quod ſi inſuper chorda DF eſt diviſa in partes infinite parvas, & æquales, & DH, DG ſunt una, duæ &c.

in inſ. partes infiniteſimæ, eadem differentiarum ſumma æqualis erit $\frac{DF^3}{3DR}$. Denominata autem diametro m, & chorda $\frac{m}{r}$, erit eadem ſumma $= \frac{m^2}{3r^3}$.

Corollarium V.

Patet differentias linearum DR, HS, GT &c. a diametro DR eſſe o, CH, BG, AF; erunt ergo hæ lineæ, ut quadrata chordarum correſpondentium o, DH, DG, DF, & earum ſumma æqualis erit ſummæ dictorum quadratorum per diametrum diviſæ. Quod ſi chordæ conſtituunt progreſſionem arithmeticam naturalem, & ſunt infinitæ, erit ſumma æqualis tertiæ parti producti ex chorda majori in differentiam majorem; hoc eſt ſi chorda major ſit DF, differentia major AF ſumma differentiarum a diametro æqualis erit $\frac{AF \times DF}{3}$.

Corollarium VI.

Hinc dato circulo DFR (Fig. 12.), & qualibet chorda DI, determinari poterunt infinita puncta curvæ parabolicæ, cuius parameter ſit circuli diameter, & ordinata major circuli chorda. Nam ſit DI diviſa in partes infinite parvas, & æquales; fiant DH, DG, DF

æqua-

æquales uni, duabus, tribus &c. infiniteſimis; ducantur AF, BG, CH parallelæ diametro, & EH, OG, MF parallelæ tangenti, quæ producantur in i, l, o, donec Ei, Ol, Mo ſint æquales chordis DH, DG, DF: dico puncta i, l, o eſſe in curva parabolica; erunt enim $DE : DO = Ei^2 : Ol^2$, $DO : DM = Ol^2 : Mo^2$, adeoque erunt abſciſſæ, ut quadrata ordinatarum. Poteſt etiam quælibet chorda DI ſupponi utcumque diviſa.

Corollarium VII.

Sit chorda DI latus quadrati circulo inſcripti, & inveniantur puncta curvæ parabolicæ infinita modo, quo diximus in Corollario præcedenti; erit ſumma ordinatarum ex dictis punctis ad axem $= \frac{ZS^2}{2} = \frac{DI^2}{2}$; ſed $DI = \sqrt{\frac{1}{2}DR^2}$, $\frac{DI^2}{2} = \frac{DR^2}{4}$; ergo ſumma dictarum ordinatarum $= \frac{DR^2}{4}$, eſt æqualis quadrato radii; ſed area parabolæ $DZS = \frac{DR\sqrt{\frac{1}{2}DR^2}}{3}$; ergo ſumma ordinatarum dicto modo ductarum in parabola DZS major eſt eiuſdem area. Eſt autem ſumma linearum in ſpatio $IFDXT = DR\sqrt{\frac{1}{2}DR^2} - \frac{DR\sqrt{\frac{1}{2}DR^2}}{6} =$

$\frac{5DR\sqrt{\frac{1}{2}DR^2}}{6}$; & ſumma linearum in ſpatio DSTX $=$ $\frac{DR\sqrt{\frac{1}{2}DR^2}}{2}$; ergo, hac dempta a prima, ſumma linearum in quadrante circuli DFIS $= \frac{DR\sqrt{\frac{1}{2}DR^2}}{3}$ æqualis eſt areæ parabolæ DZS.

Corollarium VIII.

Sit chorda circuli diameter ipſa DR (Fig. 13.) ſitque parabola DZR ſuperiori modo deſcripta; erit ſumma infinitarum ordinatarum in parabola $= \frac{DR^2}{2}$ æqualis dimidio quadrato circumſcripto; ſed area parabolæ æqualis eſt $\frac{2DR^2}{3}$; ergo area parabolæ major eſt ſumma dictarum ordinatarum. Eſt autem etiam ſumma linearum ex punctis H, G, F &c. ad tangentem RZ ductarum, quæ ſint parallelæ diametro, æqualis $\frac{2DR^2}{3}$; ergo huiuſmodi ſumma æqualis eſt areæ parabolæ.

Corollarium IX.

Cum ſumma differentiarum (ſuppoſita diametro DR pro circuli chorda) linearum HS, GT &c. a diame-

tro DR ſit æqualis $\frac{DR^2}{3}$, & ſumma linearum HS, DR &c. in infin. ſit æqualis $\frac{2DR^2}{3}$; patet ſummam differentiarum o, HC, BG &c. in infinitum, una cum ſumma linearum DR, HS, GT &c. in infin. æqualem eſſe quadrato circumſcripto.

Scholion I.

Cum ſummæ infinitarum linearum ſupra traditæ Coroll. VII. VIII. IX. exhibeant quantitates ſuperficiales, ex earum comparatione cum nonnullis ſuperficiebus, ita ut iſtis aliquando majores, aliquando minores, aliquando æquales dicantur; nihil inde ſequitur contra definitionem quantitatum homogenearum.

Scholion II.

Gravia iuxta hypotheſim Galilæanam deſcendunt motu uniformiter accelerato, ita ut temporibus æqualibus percurrant ſpatia, quæ ſint, ut numeri impares, & ſpatiorum ſummæ in fine cuiuſque temporis ſint, ut quadrata temporum, item, ut quadrata velocitatum. Ex his igitur, quæ diximus in Theorematis V. & VI., eorumque Corollariis conſtat; quod ſi grave aliquod deſcendit per DR (Fig. 11.), & tempus deſcenſus per totam

DR,

DR, ſeu velocitas in fine huius ſpatii acquiſita, ſupponitur deſignata ipſa DR; hac diviſa in partes infinite parvas, & æquales, vel etiam finitas, factiſque DH, DG, DF æqualibus uni, duabus, tribus partibus ipſius DR, eædem hæ partes exprimunt tempora cuiuslibet deſcenſus, item velocitates in fine cuiuſque ſpatii acquiſitas: lineæ CH, BG, AF denotant ſummam ſpatiorum in fine cuiuſque temporis: differentiæ vero linearum, HC a BG, BG ab AF indicant ſpatia ſingulis temporibus percurſa; nam $HC : BG = DH^2 : DG^2$; adeoque ſummæ ſpatiorum, ut quadrata temporum, ac velocitatum, & $AF - BG : BG - HC = DF^2 - DG^2 : DG^2 - DH^2 = 5 : 3$; $BG - HC : HC = DG^2 - DH^2 : DH^2 = 3 : 1$, ac propterea ſpatia ſingulis temporibus percurſa, ut numeri impares.

Scholion III.

Hinc dato tempore, quo grave ex altitudine data deſcendit, facile definientur ſpatia, quæ ſingulis iſtius temporis partibus confecit hoc modo. Exprimat DA (Fig. 14.) ſpatium toto tempore confectum, & DI totum tempus; (debet autem DI eſſe vel æqualis, vel major linea DA ſpatium deſignante) ſi DI major eſt DA, in A erigatur perpendicularis indefinita AI, quæ determinetur centro D, intervallo DI, deſcribendo arcum

ſe-

ſecantem AI in aliquo puncto I; a puncto D ad I ducatur DI; in I elevetur perpendicularis IS; producatur DA, donec concurrat cum IS in S; ſupra DS tamquam diametro deſcribatur circulus tranſiens per tria puncta D, I, S; recta DI dividatur in partes æquales; fiatque DH æqualis uni, DG duabus, DF tribus partibus ipſius DI; ex punctis H, G, F ducantur perpendiculares AB, GC, FE; erunt DB, BC, CE, EA, DA ſpatia prima, ſecunda, tertia, quarta parte temporis, & toto tempore DI percurſa. Quod ſi DI = DA, ſumpta DA tamquam diametro, deſcribatur circulus; dividatur DA in partes æquales, fiat DH æqualis uni parti ipſius diametri, & cætera, ut ſupra. Similiter dato tempore, quo grave in medio non reſiſtente per ſpatium datum deſcendit, determinabitur tempus, quo aliud ſpatium datum in eodem medio conficiet. Sint enim ſpatia data DA, & DE; erigatur ut ſupra perpendicularis AI; determinetur in AI punctum, ad quod ducenda DI exprimens tempus per DA; in I elevetur perpendicularis IS; & deſcribatur circulus; in E fiat EF normalis, & conjungatur DF, hæc deſignabit tempus per DE; quod ſi DE major ſit DA, major etiam ſit ratio DI : DA. Horum, & ſimilium demonſtratio patet ex dictis.

Scholion IV.

Sit ſemicirculus DIM (Fig. 15.) cuius diameter DM ſit diviſa in partes finitas, vel infinite parvas, & æquales. Sint DH, DG, DF &c. æquales uni, duabus, tribus partibus diametri. Ducantur perpendiculares HX, GQ, FP &c. ſi mobile aliquod movebitur motu compoſito, ex horizontali, & uniformiter accelerato; ita tamen, uti ſpatia horizontalia ſingulis temporibus æqualibus a mobili percurſa adæquent differentias perpendicularium o, HX, GQ, FP: dico, quod dictum mobile in fine cuiuslibet temporis erit in circumferentia circuli. Huius rei veritas partim patet ex dictis, partim ex hypotheſi; etenim cum in D perpendicularis ſit = o, erit, exiſtente mobili in D, differentia = XH = DC; ergo ex hypotheſi motu horizontali percurret DC; ſed eodem tempore ex dictis motu accelerato percurrere debet HC; ergo in fine primi temporis mobile erit in H; pariter differentia inter HX, & GQ = Hi; ergo ſecundo tempore motu horizontali transferetur in i; ſed motu accelerato deſcendet in G; ergo in fine ſecundi temporis erit in puncto G circumferentiæ; cumque ſit ita ſemper; ergo mobile in fine cuiuſque temporis erit in circuli circumferentia.

Scho-

Scholion V.

Facile patet etiam ex dictis, quod mobile; quod movetur motu compoſito ex æquabili ſecundum lineam horizonti parallelam, & uniformiter accelerato, debet parabolam deſcribere; nam fiat Dc : Db = DH : DG, & ducantur ch, bg parallelæ, & æquales CH, BG, erit ch : bg, ut quadratum temporis DH ad quadratum temporis DG; ſed ſpatia Dc, Db eadem velocitate percurſa ſunt, ut tempora; tempora autem ſunt eadem, quæ ſpatiorum ch, bg; ergo Dc : Db = DH : DG; ac proinde ch : bg = $Dc^2 : Db^2$. Sunt igitur abſciſſæ ch, bg, ſeu DX, DQ, ut quadrata ordinatarum Dc, Db, ſeu Xh, Qg; ergo mobile, quod movetur, ut ab initio fuit dictum, parabolam deſcribit.

THEOREMA VII.

Dato circulo ASXC (Fig. 16.) ſi deſcribitur parabola AOQ, cuius axis AX, tranſiens per duo alia quælibet circuli puncta S, & C, rectangulum ex linea compoſita ex ſemiordinata circuli, & ſemiordinata parabolæ in interceptam inter curvam parabolicam, & circularem æquale eſt rectangulo ex ſegmentis axis a vertice ad ordinatam aſſumptam, & ab eadem ordinata

K ad

ad ordinatam communem circulo, & parabolæ; nimirum rectangulum EVT, ſeu ZTV eſt æquale rectangulo ARD, & rectangulum FQY, ſeu OYQ eſt æquale rectangulo APD.

Demonſtratio.

Eſt $AR \times AB$ (AB eſt parameter) $= RT^2$, & $AR \times RX = RV^2$; ergo eſt $AR \times RX : AR \times AB = RV^2 : RT^2$; hoc eſt $RX : AB = RV^2 : RT^2$; & dividendo $RX - AB : AB = RV^2 - RT^2 : RT^2$; ſed $RX - AB = RD$, ob $AB = DX$, $RV^2 - RT^2 = EVT = ZTV$; ergo $RD : AB = EV \times VT : RT^2$; eſt autem $RT^2 = AB \times AR$; ergo $RD : AB = EV \times VT : AB \times AR$; & $RD \times AB \times AR = EV \times VT \times AB$; ergo, utroque producto diviſo per AB, $EV \times VT = ZT \times TV = AR \times RD$. Eodem modo demonſtrabitur rectangulum $AP \times PD = FQ \times QY = OY \times YQ$. Q. e. d.

Scholion.

Poteſt parabolæ tranſeuntis per punctum A circuli ASXC parameter AB eſſe minor diametro circuli AX, ut fig. 16., huic eſſe æqualis, ut fig. 17., illo eſſe major, ut fig. 18. In primo caſu parabola tranſiens per verticem A circulum ſecat in aliis duobus punctis, ut S,

& C;

& C, in reliquis caſibus nulli alteri circuli puncto occurrit, ſed tantum, quo major eſt parameter diametro, eo magis curva parabolica divergit a circulari. Quid contingat in primo caſu jam oſtendimus in ſuperiori Theoremate; ſupereſt ergo examinandum quid dicendum ſit in aliis caſibus. Sit igitur.

THEOREMA VIII.

Sint omnia, ut ſupra: dico rectangulum ex linea compoſita ex ſemiordinata parabolæ, & ſemiordinata circuli, ſuppoſito parametro æquali diametro, in interceptam inter curvam parabolicam, & circularem æqualem eſſe quadrato abſciſſæ a vertice ad ordinatam aſſumptam. Hoc eſt rectangulum $ZT \times TV = AR^2$; $FQ \times QY = AP^2$. (Fig. 17.)

Demonſtratio.

$AR \times AB = RT^2$; $AR \times RX = RV^2$; ergo $AB : RX = RT^2 : RV^2$; & dividendo $AB - RX : RX = RT^2 - RV^2 : RV^2$; ſed, quia per hyp. $AB = AX$, eſt $AB - RX = AR$; ergo $AR : RX = RT^2 - RV^2 : RV^2$; eſt autem $RT^2 - RV^2 = ZT \times TV$, & $RV^2 = AR \times RX$; ergo $AR : RX = ZT \times TV : AR \times RX$; & $AR \times AR \times RX = RX \times ZT \times TV$; hoc eſt AR^2

 $= ZT$

$= ZT \times TV$. Eodem modo demonſtrabitur $FQ \times QY = AP^2$. Q. e. d.

THEOREMA IX.

Sint omnia, ut in Theoremate VII., & VIII., & ſit parameter AB (Fig. 18.) major diametro AX; dico rectangulum ex linea compoſita ex ſemiordinata circuli, & parabolæ in interceptam inter curvam parabolicam, & circularem æquale eſſe differentiæ rectanguli ex parametro in abſciſſam ab ordinata aſſumpta determinatam a rectangulo ex ſegmentis axis per eandem ordinatam determinatis; hoc eſt $ZT \times TV = AB \times AR - AR \times RX$; & $FQ \times QY = AB \times AP - AP \times PX$.

Demonſtratio.

$AR \times AB = RT^2$; $AR \times RX = RV^2$; ergo $AB : RX = RT^2 : RV^2$; & dividendo $AB - RX : RX = RT^2 - RV^2 : RV^2$; ſed eſt $RT^2 - RV^2 = ZT \times TV$, & $RV^2 = AR \times RX$; ergo $AB - RX : RX = ZT \times TV : AR \times RX$; ergo $AB \times AR - RX \times AR = ZT \times TV$. Eodem modo demonſtrabitur $FQ \times QY = AB \times AP - AP \times PX$. Q. e. d.

Scho-

Scholion I.

Si AX (Fig. 16. 17. 18.) axis circuli, & parabolæ dicatur m, quælibet AR exprimi poterit per $\frac{m}{r}$; & AB parabolæ parameter axi AX vel æqualis, vel multiplex, vel ſubmultiplex deſignari poterit per $\frac{m}{n}$. Suppoſita ergo qualibet AR diviſa in partes infinite parvas, & æquales, & a qualibet parte ad curvam parabolicam, & circularem ductis ordinatis, ſingula quadrata ſemiordinatarum parabolæ exhibebit hæc formula univerſalis $\frac{m^2}{rn} - \frac{o}{n} + \frac{m^2}{rn} - \frac{m}{n} + \frac{m^2}{rn} - \frac{2m}{n} + \frac{m^2}{rn} - \frac{3m}{n}$ &c. Ex huius formulæ inſpectione conſtat quadratum $\frac{m^2}{rn}$ eſſe conſtans in ſingulis terminis, & in qualibet ſumma quadratorum ſemiordinatarum parabolæ tot eſſe huiuſmodi quadrata, quot ſunt partes in abſciſſa AR, quarum ſumma exprimitur per $\frac{m}{r}$; ergo ſumma infinitorum quadratorum $\frac{m^2}{rn} = \frac{m^2}{rn} \times \frac{m}{r} = \frac{m^3}{r^2 n}$. Pariter ex eiuſdem formulæ conſideratione patet terminos $-\frac{o}{n} - \frac{m}{n} - \frac{2m}{n}$ &c. con-

conſtituere progreſſionem arithmeticam infinitam numerorum naturalium ex tot terminis conſtantem, quot partes continet $\frac{m}{r}$, quorum terminus maximus eſt $\frac{m}{r}$, & quilibet eſt ductus in $\frac{m}{n}$; ſed ſumma infinitorum numerorum naturalium, quorum maximus terminus $\frac{m}{r} = \frac{m^2}{2r^2}$; ergo hac ducta in $\frac{m}{n}$ habebitur ſumma infinitorum terminorum $\frac{o}{n} + \frac{m}{n} + \frac{2m}{n}$ &c. $= \frac{m^3}{2r^2n}$; ergo ſumma infinitorum quadratorum ſemiordinatarum parabolæ $= \frac{m^3}{r^2n} - \frac{m^3}{2r^2n} = \frac{m^3}{2r^2n}$.

Scholion II.

Suppoſita adhuc AX (Fig. 16. 17. 18.) = m, & quælibet AR $= \frac{m}{r}$, & diviſa in partes infinite parvas, & æquales, erit quælibet RX $= m - \frac{m}{r}$; ergo erit rectangulum AR $\times$ RX $= \overline{\frac{m}{r} - o} \times \overline{m - \frac{m}{r} + o}$; rectangulum ex recta AR diminuta parte infiniteſima in rectam RX parte infiniteſima auctam $= \overline{\frac{m}{r} - 1} \times$

$\frac{m}{}$ —

$\overline{m - \frac{m}{r} + 1}$; idem dicatur cum proportione de reliquis rectangulis ex ſegmentis diametri. Si actu igitur multiplicabuntur hæc latera, ut habeantur ſingula rectangula, obtinebitur hæc formula exprimens infinita rectangula ſegmentorum diametri, nimirum

$$\frac{m^2}{r} - \frac{m^2}{r^2} + 0 - 0 - 0$$
$$+ \frac{m^2}{r} - \frac{m^2}{r^2} + \frac{2m}{r} - m - 1$$
$$+ \frac{m^2}{r} - \frac{m^2}{r^2} + \frac{4m}{r} - 2m - 4$$
$$+ \frac{m^2}{r} - \frac{m^2}{r^2} + \frac{6m}{r} - 3m - 9 \text{ \&c.},$$

quorum ſumma æqualis erit $\frac{m^3}{2r^2} - \frac{m^3}{3r^3}$; nam ſumma omnium $\frac{m^2}{r} = \frac{m^2}{r} \times \frac{m}{r} = \frac{m^3}{r^2}$; ſumma omnium $-\frac{m^2}{r^2} = -\frac{m^2}{r^2} \times \frac{m}{r} = -\frac{m^3}{r^3}$; ſumma terminorum $0 + \frac{2m}{r} + \frac{4m}{r} + \frac{6m}{r}$ &c. $= + \frac{m^2}{r^2} \times \frac{m}{r} = + \frac{m^3}{r^3}$; ſumma omnium terminorum $- 0 - m - 2m - 3m$ &c. $= - \frac{m^2}{2r^2} \times m = - \frac{m^3}{2r^2}$; ſumma denique omnium terminorum $- 0 - 1 - 4 - 9$ &c. $= - \frac{m^3}{3r^3}$; ergo ſumma om-

omnium rectangulorum æqualis eſt $\frac{m^3}{r^2} - \frac{m^3}{r^3} + \frac{m^3}{r^3} - \frac{m^3}{2r^2} - \frac{m^3}{3r^3}$, quæ ad ſimpliciores terminos redacta eſt æqualis $\frac{m^3}{2r^2} - \frac{m^3}{3r^3}$.

Corollarium I.

Si abſciſſa AR ponitur æqualis radio, æqualis parametro AB, (Fig. 16.) tunc $\frac{m}{r}$, & $\frac{m}{n} = \frac{m}{2}$; erit ergo ſumma omnium quadratorum ordinatarum parabolæ DAC, quæ eſt $\frac{m^3}{2r^2n}$, æqualis $\frac{m^3}{16}$; ſed ſumma omnium quadratorum ordinatarum quadrantis DAVC eſt æqualis $\frac{m^3}{12}$; ergo, ſi ab hac demitur prima, ſumma omnium rectangulorum ex lineis compoſitis ex ſemiordinatis circuli, & parabolæ in interceptas inter curvam parabolicam ATC, & circularum AVC eſt æqualis $\frac{m^3}{48}$. Eſt ergo ſumma quadratorum ordinatarum quadrantis ad ſummam quadratorum ordinatarum parabolæ $= 4 : 3$; & utraque ſumma ad ſummam differentiarum inter ſe invicem $= 4 : 1$, & $3 : 1$.

Corol-

Corollarium II.

Suppoſita AR $= \frac{m}{r} =$ AX $=$ m, & parametro AB $= \frac{m}{n} = \frac{m}{2}$, (Fig. 16.) ſumma omnium quadratorum ſemiordinatarum parabolæ ad axem AX $= \frac{m^3}{2r^2n}$ eſt æqualis $\frac{m^3}{4}$; ſed ſumma omnium quadratorum ſemiapplicatarum parabolæ ad ſemiaxem AD eſt æqualis $\frac{m^3}{16}$; ergo ſumma quadratorum ſemiapplicatarum parabolæ ad ſemiaxem DX eſt æqualis $\frac{3m^3}{16}$. Eſt autem ſumma quadratorum ordinatarum quadrantis DXC æqualis $\frac{m^3}{12}$; ergo ſumma infinitorum rectangulorum fqy, FQY eſt æqualis $\frac{5m^3}{48}$.

Corollarium III.

Cum (Fig. 16.) ſumma quadratorum ſemiordinatarum ad axem AX parabolæ SAC ſit æqualis $\frac{m^3}{4}$; ſumma exceſſuum quadratorum ſemiordinatarum circuli ſupra quadrata ſemiordinatarum parabolæ ad axem AD,

una cum ſumma exceſſuum quadratorum ſemiordinatarum parabolæ ſupra quadrata ſemiordinatarum circuli ad axem DX ſit æqualis $\frac{m^3}{48} + \frac{5m^3}{48} = \frac{6m^3}{48} = \frac{m^3}{8}$; ſumma tandem quadratorum ſemiordinatarum circuli ad diametrum AX ſit æqualis $\frac{m^3}{6}$, patet primam eſſe ad ſecundam $= \frac{m^3}{4} : \frac{m^3}{8} = 2 : 1$, & ſecundam ad tertiam $= \frac{m^3}{8} : \frac{m^3}{6} = 1 : 1\frac{1}{3}$.

Corollarium IV.

Suppoſita diametro AX æquali parametro AB, (Fig. 17.) erit ſumma quadratorum ſemiordinatarum parabolæ ad axem AX $= \frac{m^3}{2}$, & ſumma quadratorum ſemiordinatarum circuli ad eundem axem $= \frac{m^3}{6}$; ergo, hac dempta a prima, erit ſumma infinitorum rectangulorum ZTV, ztu, FQY &c. $= \frac{m^3}{2} - \frac{m^3}{6} = \frac{6m^3 - 2m^3}{12} = \frac{m^3}{3}$. Hoc idem poterat immediate inferri ex Theor. VIII., in quo demonſtratur quodlibet rectangulum ZTV æquale eſſe quadrato correſpondentis abſciſſæ AR; ſuppoſita enim diametro AX = m diviſa in partes infinite par-

parvas, & æquales, ductiſque lineis ZT, zt &c. ſumma rectangulorum ZTV, ztu æqualis eſt ſummæ quadratorum numerorum naturalium, quorum maximus AX = m; hæc autem eſt æqualis $\frac{m^3}{3}$; ergo etiam illa.

Corollarium V.

Invenimus Corol. I. ſummam quadratorum ſemiordinatarum parabolæ ad axem AD radium circuli (Fig. 16.) æqualem eſſe $\frac{m^3}{16}$; & Corol. II. ſummam quadratorum ſemiordinatarum parabolæ ad axem AX diametrum circuli æqualem eſſe $\frac{m^3}{4}$; ergo illa eſt ad hanc, ut quadratum radii ad quadratum diametri, ut $\frac{1}{4}$: 1, ut ſumma exceſſuum quadratorum ex ordinatis quadrantis DAVC ſupra quadrata ordinatarum parabolæ DAC ad axem AD ad ſummam quadratorum ordinatarum eiuſdem quadrantis DAVC.

Corollarium VI.

Si ponitur (Fig. 17.) AX = AB = m, ſumma omnium quadratorum ſemiordinatarum parabolæ ad axem AX = $\frac{m^3}{2}$, ſumma omnium exceſſuum quadratorum ſemiordinatarum parabolæ ſupra quadrata ſemiordinata-

rum circuli $= \frac{m^3}{3}$, ſumma denique omnium quadratorum ſemiordinatarum circuli $= \frac{m^3}{6}$; ergo prima eſt ad ſecundam $= \frac{1}{2} : \frac{1}{3}$, ſecunda ad tertiam $= \frac{1}{3} : \frac{1}{6}$, hoc eſt ſumma rectangulorum ZTV, FQY &c. eſt media arithmetica inter primam, & ultimam ſummam.

Corollarium VII.

Suppoſita adhuc $AX = AB = m$, (Fig. 17.) ſumma quadratorum ſemiordinatarum parabolæ ad axem $AX = \frac{m^3}{2}$, ad ſemiaxem $AD = \frac{m^3}{8}$; ergo prima eſt ad ſecundam, ut quadratum diametri ad quadratum radii. Eſt autem ſumma quadratorum ordinatarum quadrantis $DAVC = \frac{m^3}{12}$, ſumma differentiarum quadratorum parabolæ ſupra quadrata quadrantis $= \frac{m^3}{24}$; ergo ſumma $\frac{m^3}{12}$ eſt media arithmetica inter ſummam $\frac{m^3}{8}$, & $\frac{m^3}{24}$.

Corollarium VIII.

Hinc, ſi (Fig. 16.) ponitur $AD = \frac{m}{2} = AB$, formula univerſalis ſummæ quadratorum ſemiordinatarum para-

parabolæ ad axem $AD = \frac{m^3}{2r^2n}$ commutatur in hanc aliam $\frac{m^3}{16}$, & formula univerſalis ſummæ rectangulorum ex ſegmentis diametri $\frac{m^3}{2r^2} - \frac{m^3}{3r^3}$ reducitur ad hanc aliam $\frac{m^3}{8} - \frac{m^3}{24} = \frac{m^3}{12}$; ergo, ſi ab hac $\frac{m^3}{12}$ ſubtrahitur $\frac{m^3}{16}$, remanet $\frac{m^3}{48}$ ſumma rectangulorum EVT, eut &c. in infin. Pariter, ſi (Fig. 17.) ſit $AD = m$, $AB = m$, formula ſuperior $\frac{m^3}{2r^2n}$ æquivalet huic alteri $\frac{m^3}{8}$, & altera $\frac{m^3}{2r^2} - \frac{m^3}{3r^3}$ huic $\frac{m^3}{8} - \frac{m^3}{24} = \frac{m^3}{12}$; ergo, ſi iſta $\frac{m^3}{12}$ demitur a prima $\frac{m^3}{8}$, ſumma rectangulorum ZTV, ztu &c. in infin. $= \frac{m^3}{24}$. Similiter, ſi (Fig. 18.) ponitur $AD = \frac{m}{2}$, $AB = 2m$, eadem formula $\frac{m^3}{2r^2n}$ eſt æqualis $\frac{m^3}{4}$, & alia $\frac{m^3}{2r^2} - \frac{m^3}{3r^3}$ eſt æqualis $\frac{m^3}{8} - \frac{m^3}{24} = \frac{m^3}{12}$; ergo, hac dempta a prima, ſumma rectangulorum ZTV, ztu &c. in infin. $= m^3$.

Corol-

Corollarium IX.

Suppoſita adhuc $AD = \frac{m}{2} = AB$, (Fig. 16.) ſi intelligitur revolvi tum parabola DAC, tum quadrans DAVC circa axem AD, ex revolutione primæ oritur conois parabolica, ſecundi emiſpherium; erit autem hoc ad illam, ut ſumma quadratorum RV, ru &c. ad ſummam quadratorum RT, rt &c.; ſed prima ſumma æqualis eſt $\frac{m^3}{12}$, ſecunda $= \frac{m^3}{16}$; ergo emiſpherium eſt ad conoidem parabolicam, ut $\frac{m^3}{12} : \frac{m^3}{16} = 4 : 3$. Erit vero eadem conois ad exceſſum emiſpherii ſupra ipſam conoidem, ut ſumma quadratorum RT, rt, ad differentiam eorundem à quadratis RV, ru &c.; ſed prima eſt $= \frac{m^3}{16}$, ſecunda $= \frac{m^3}{48}$; ergo conois parabolica eſt ad exceſſum emiſpherii ſupra ipſam conoidem, ut $\frac{m^3}{16} : \frac{m^3}{48} = \frac{3m^3}{48} : \frac{m^3}{48} = 3 : 1$.

Corollarium X.

Stante adhuc $AB = AD = \frac{m}{2}$, (Fig. 16.) & $AX = m$, conois parabolica orta ex revolutione parabolæ AXN circa

circa axem AX erit ad ſphæram ex revolutione ſemicirculi ACX genitam, ut $\frac{m^3}{4} : \frac{m^3}{6}$, & hæc ad exceſſum conoidis ſupra ipſam ſphæram, ut $\frac{m^3}{6} : \frac{m^3}{12}$; ſe habebunt ergo inter ſe invicem, ut $\frac{m^3}{4} : \frac{m^3}{6} : \frac{m^3}{12}$, ſeu, ut $\frac{3m^3}{12} : \frac{2m^3}{12} : \frac{m^3}{12}$, hoc eſt, ut $3 : 2 : 1$. Sed in eadem ratione ſunt etiam cylindrus circumſcriptus, ſphæra ACXS inſcripta, & conus eiuſdem baſis, & altitudinis cum cylindro; ergo conois dicto modo genita æqualis eſt cylindro circumſcripto, & exceſſus conoidis ſupra ſphæram æqualis eſt cono eiuſdem baſis, & altitudinis cum cylindro.

Corollarium XI.

Exceſſus portionis conoidis SMXNC ſupra emiſpherium CXS erit ad exceſſum emiſpherii SAC ſupra partem conoidis CAS $= 5 : 1$, patet ex Corol. præced., & IV. Quare erit primus exceſſus ad conoidem CAS $= 5 : 3$, & hic ad ſecundum exceſſum $= 3 : 1$, adeoque erit conois CAS media arithmetica inter utrumque exceſſum.

Corol-

Corollarium XII.

Sit modo (Fig. 17.) parameter AB = AX = m, erit conois parabolica ex revolutione parabolæ AXN circa axem AX ad ſphæram genitam a ſemicirculo ACX $= \frac{m^3}{2} : \frac{m^3}{6}$; & ſphæra ad exceſſum conoidis ſupra ipſam ſphæram $= \frac{m^3}{6} : \frac{m^3}{3}$, hoc eſt erit conois ad exceſſum eiuſdem ſupra ſphæram $\frac{3m^3}{6} : \frac{2m^3}{6}$, & hic ad ſphæram $= \frac{2m^3}{6} : \frac{m^3}{6}$; eſt ergo conois ad exceſſum eiuſdem ſupra ſphæram, ut cylindrus ſphæræ ACXS circumſcriptus ad ſphæram inſcriptam, & idem exceſſus ad ſphæram, ut ſphæra ad conum eiuſdem baſis, & altitudinis cum cylindro.

THEOREMA X.

Dato circulo ASXC, (Fig. 19.) & qualibet hyperbola AOQ, cujus axis ſit AX communis circulo, & hyperbolæ, dico rectangulum ex linea compoſita ex ſemiordinata circuli, & hyperbolæ in interceptam inter curvam circularem, & hyperbolicam æquale eſſe differentiæ rectanguli ex linea compoſita ex aſſe tranſverſo,

&

& abſciſſa in abſciſſam diviſi per quadratum denominatoris rationis eiuſdem axis tranſverſi ad axem coniugatum a rectangulo ex ſegmentis diametri circuli: hoc eſt, ſi $RT^2 > RV^2$, $ZT \times TV = \frac{BR \times AR}{D^2} - RX \times AR$, & ſi $RT^2 < RV^2$; $ZV \times VT = RX \times AR - \frac{BR \times AR}{D^2}$.

Demonſtratio.

Sit AB axis tranſverſus, HI coniugatus, & denominator rationis $AB : HI = D$, (exprimat D numerum quemcumque integrum, vel fractum), erit $HI = \frac{AB}{D}$.

Cum vero hyperbolæ latus rectum, ſeu parameter ſit tertia proportionalis axi tranſverſo, & coniugato, erit latus rectum $= \frac{AB}{D^2}$. Hoc ſuppoſito: ex natura hyperbolæ, eſt $AB : \frac{AB}{D^2} = BR \times AR : RT^2$; ſed $RT^2 = RV^2 + ZT \times TV = AR \times RX + ZT \times TV$; ergo $AB : \frac{AB}{D^2} = BR \times AR : AR \times RX + ZT \times TV$; ergo $\frac{AB \times BR \times AR}{AB \times D^2} = AR \times RX + ZT \times TV$, hoc eſt $\frac{BR \times AR}{D^2} - AR \times RX = ZT \times TV$; ex iiſdem prin-

M cipiis

cipiis conſtat, ſuppoſito $RT^2 < RV^2$, eſſe $ZV \times VT = RX \times AR - \frac{BR \times AR}{D^2}$. Q. e. d.

Corollarium I.

Si hyperbola AOQ erit æquilatera, & axis tranſverſus $AB = AX$ diametro circuli, tunc quodlibet rectangulum $ZT \times TV$ erit æquale duplo quadrato correſpondentis abſciſſæ $= 2AR^2$; nam $BR \times AR = RT^2$, & $AR \times RX = RV^2$; ergo $BR : RX = RT^2 : RV^2$; & dividendo $BR - RX : RX = RT^2 - RV^2 : RV^2$; ſed $BR = AX + AR$, $RX = AX - AR$, $RT^2 - RV^2 = ZT \times TV$, $RV^2 = AR \times RX$; ergo $2AR : RX = ZT \times TV : AR \times RX$, & $2AR^2 = ZT \times TV$.

Corollarium II.

Si per circuli punctum tranſit hyperbola æquilatera, cuius axis tranſverſus ſit æqualis diametro circuli, & parabola, cuius parameter pariter ſit æqualis diametro eiuſdem circuli, erit ſumma infinitorum rectangulorum ex lineis compoſitis ex ſemiordinatis circuli, & hyperbolæ in interceptas inter curvam circularem, & hyperbolicam ad ſummam infinitorum rectangulorum ex lineis compoſitis ex ſemiordinatis circuli, & parabolæ in interceptas inter curvam circularem, & parabolicam,

ut

ut 2 : 1 ; nam ex Corol. præc. quodlibet rectangulum ſupradictum hyperbolæ eſt æquale duplo quadrato correſpondentis abſciſſæ ; & ex Theor. VIII. quodlibet rectangulum parabolæ eſt æquale quadrato correſpondentis abſciſſæ.

Corollarium III.

Si ſemiordinata hyperbolæ major eſt ſemiordinata circuli, quodlibet quadratum ſemiordinatæ primæ eſt æquale quadrato ſemiordinatæ ſecundi, & rectangulo ex linea compoſita ex ſemiordinata hyperbolæ, & circuli in interceptam inter curvam circularem, & hyperbolicam, nimirum $RT^2 = RV^2 + ZT \times TV = AR \times RX + ZT \times TV$; ſed $AR \times RX + ZT \times TV = \frac{BR \times AR}{D^2}$; ergo $RT^2 = \frac{BR \times AR}{D^2}$.

Corollarium IV.

Viceverſa quodlibet quadratum ſemiordinatæ hyperbolæ æquale eſt quadrato ſemiordinatæ circuli, dempto rectangulo ex linea compoſita ex ſemiordinata primæ, & ſecundi in interceptam inter curvam circularem, & hyperbolicam, ſi ſemiordinata prima minor eſt ſecunda; hoc eſt $RT^2 = RV^2 - ZV \times VT = AR \times RX -$

$ZV \times TV$; ſed $RX \times AR - ZV \times TV = \frac{BR \times AR}{D^2}$;
ergo $RT^2 = \frac{BR \times AR}{D^2}$.

Scholion:

Suppoſita diametro circuli $AX = m$; qualibet $AR = \frac{m}{r}$, axe tranſverſo $= a$, denominatore rationis axis tranſverſi ad coniugatum $= d$; & inſuper ſuppoſita AR diviſa in partes infinite parvas, & æquales, erit $\frac{BR \times AR}{D^2} = RT^2 = \overline{\frac{a}{d} + \frac{m}{rd} - \frac{o}{d}} \times \overline{\frac{m}{rd} - \frac{o}{d}}$, & $\overline{\frac{BR - 1}{d}} \times \overline{\frac{AR - 1}{d}} = \overline{\frac{a}{d} + \frac{m}{rd} - \frac{1}{d}} \times \overline{\frac{m}{rd} - \frac{1}{d}}$ &c.; ergo ſumma omnium quadratorum ſemiordinatarum hyperbolæ ad rectam AR æqualis erit

$$\frac{am}{rd^2} + \frac{m^2}{r^2d^2} - o - o + o$$
$$+ \frac{am}{rd^2} + \frac{m^2}{r^2d^2} - \frac{2m}{rd^2} - \frac{a}{d^2} + \frac{1}{d^2}$$
$$+ \frac{am}{rd^2} + \frac{m^2}{r^2d^2} - \frac{4m}{rd^2} - \frac{2a}{d^2} + \frac{4}{d^2}$$
$$+ \frac{am}{rd^2} + \frac{m^2}{r^2d^2} - \frac{6m}{rd^2} - \frac{3a}{d^2} + \frac{9}{d^2} \text{ \&c.};$$

hoc

hoc eſt æqualis erit $\frac{am^2}{r^2d^2} + \frac{m^3}{r^3d^2} - \frac{m^3}{r^3d^2} - \frac{am^2}{2r^2d^2} +$

$\frac{m^3}{3r^3d^2} = \frac{am^2}{2r^2d^2} + \frac{m^3}{3r^3d^2}$.

Corollarium V.

Si hyperbola erit æquilatera, & axis tranſverſus AB æqualis AX, erit ſumma omnium quadratorum ſemiordinatarum eiuſdem $= \frac{m^3}{2r^2} + \frac{m^3}{3r^3}$.

Corollarium VI.

Sit AXN hyperbola æquilatera, cuius axis AB æqualis diametro AX, & revolvatur una cum ſemicirculo circa axem AX, erit conois hyperbolica genita ex revolutione primæ ad ſphæram ex revolutione ſecundi $= \frac{5m^3}{6} : \frac{m^3}{6}$, & conois eadem ad eiuſdem exceſſum ſupra ſphæram $= \frac{5m^3}{6} : \frac{4m^3}{6}$; ergo ſunt inter ſe, ut 5 : 4 : 1; ſed, ſi per punctum A tranſiret parabola, cuius parameter = AX, conois genita ex hac parabola eſſet ad exceſſum eiuſdem ſupra ſphæram = 3 : 2, & hic exceſſus ad ſphæram = 2 : 1 per Corol. XII. Theor. IX.; ergo conois hyperbolica, exceſſus eiuſdem ſupra ſphæ-

ram,

ram, conois parabolica, exceſſus eiuſdem ſupra ſphæram, & ſphæra, eſſent in progreſſione arithmetica continua, nimirum $= 5 : 4 : 3 : 2 : 1$.

THEOREMA XI.

Dato circulo ASXC, (Fig. 20.) & Ellypſi AFXY, quarum figurarum axis communis ſit AX, erit rectangulum ex linea compoſita ex ſemiordinata circuli, & ellypſis, in interceptam inter curvam circularem, & ellypticam æquale rectangulo abſciſſarum ab utroque vertice diminuto eodem rectangulo diviſo per quadratum denominatoris rationis axis tranſverſi ad conjugatum.

Demonſtratio.

Sit AX axis tranſverſus ellypſis, FY conjugatus, & ſit denominator rationis $AX : FY = D$; erit $FY = \frac{AX}{D}$; cum autem tertia proportionalis axi tranſverſo, & conjugato ellypſis ſit latus rectum, ſeu parameter, erit parameter $= \frac{AX}{D^2}$; ſed ex natura ellypſis $AX : \frac{AX}{D^2} = AR \times RX : RV^2$; ergo $RV^2 = \frac{AR \times RX}{D^2}$; ſed ulterius $RT^2 = AR \times RX$; ergo $RT^2 - RV^2 = AR \times RX - \frac{AR \times RX}{D^2}$, hoc eſt $ZT \times TV = AR \times RX - \frac{AR \times RX}{D^2}$. Q. e. d.

Scho-

Scholion I.

Suppoſita $AX = m$, qualibet $AR = \frac{m}{r}$, quadrato denominatoris rationis axis tranſverſi ad conjugatum $= d^2$, erit ſumma infinitorum quadratorum ellypſis $RV^2 = \frac{m^3}{2r^2d^2} - \frac{m^3}{3r^3d^2}$.

Corollarium I.

Per Scholion Theor. IX. ſumma infinitorum rectangulorum ex ſegmentis diametri circuli eſt æqualis $\frac{m^3}{2r^2} - \frac{m^3}{3r^3}$; ſed ex hoc Theoremate quodlibet rectangulum $ZT \times TV = AR \times RX - \frac{AR \times RX}{D^2}$; ergo ſumma infinitorum rectangulorum $ZT \times TV = \frac{m^3}{2r^2} - \frac{m^3}{3r^3} - \frac{m^3}{2r^2d^2} + \frac{m^3}{3r^3d^2}$.

Corollarium II.

Ex revolutione ſemicirculi ACX, & ſemiellypſis AYX circa axem AX oritur ſphæra, & ſphæroides, erit autem illa ad hanc, ut ſumma quadratorum RT,

rt &c.

rt &c. ad ſummam quadratorum RV, ru, & illa eadem ad exceſſum eiuſdem ſupra ſphæroidem, ut ſumma quadratorum RT, rt ad differentiam eorundem a quadratis RV, ru, hoc eſt $= \frac{m^3}{2} - \frac{m^3}{3} : \frac{m^3}{2d^2} - \frac{m^3}{3d^2} = \frac{m^3}{6} : \frac{m^3}{6d^2} = 1 : \frac{1}{d^2}$, & $\frac{m^3}{2} - \frac{m^3}{3} : \frac{m^3}{2} - \frac{m^3}{3} - \frac{m^3}{2d^2} + \frac{m^3}{3d^2} = \frac{m^3}{6} : \frac{m^3}{6} - \frac{m^3}{6d^2} = 1 : 1 - \frac{1}{d^2}$.

Scholion II.

Eodem ratiocinio determinari poteſt etiam proportio, quam habet ſoliditas cuiuslibet ſegmenti ſphærici ad ſoliditatem ſegmenti ſphæroidalis, vel conoidalis, ſive hyperbolici, ſive parabolici, quorum vertex idem, & idem axis, ut ſatis eſt per ſe clarum; imo determinari poteſt etiam proportio ſoliditatis tum integræ ſphæræ, tum ſegmenti eiuſdem ad ſoliditatem nonnullorum corporum ſphæræ circumſcriptorum, vel inſcriptorum. Atque, ut huius rei exemplum aliquod aſſignemus. Proponamus primo inveſtigandam proportionem ſoliditatis ſphæræ ad ſoliditatem coni æquilateri inſcripti, erit prima ad ſecundam, ut ſumma quadratorum RT^2, rt^2 &c. (Fig. 21.) in infin. ad ſummam quadratorum RV^2, ru^2 &c. in infin.; ſed prima ſumma $= \frac{m^3}{6}$, ſecun-

da

da $= \frac{3m^3}{64}$, quia quadrata RV^2, ru^2 &c. ſunt quadrata numerorum naturalium; ergo ſoliditas ſphæræ eſt ad ſoliditatem coni æquilateri inſcripti $= 32 : 9$. Proponamus ſecundo determinandam proportionem ſoliditatis ſegmenti ſphærici AESGCTA ad conum æquilaterum inſcriptum; erit prima ad ſecundum, ut ſumma quadratorum RT^2, rt^2 in ſegmento ſphærico ad ſummam quadratorum RV^2, ru^2 in cono æquilatero; ſed prima ſumma æqualis $\frac{9m^3}{64}$, ſecunda $\frac{3m^3}{64}$; ergo ſoliditas ſegmenti ſphærici eſt ad ſoliditatem coni æquilateri $= 3 : 1$, adeoque ſegmentum ſphæricum circumſcriptum cono æquilatero æquale eſt cylindro eiuſdem baſis, & altitudinis cum cono. Proponamus tertio inveniendam proportionem ſoliditatis cylindri circumſcripti ad ſphæram inſcriptam; erit ſumma omnium quadratorum æqualium cylindri $= \frac{m^3}{4}$, & ſumma omnium quadratorum ſphæræ $= \frac{m^3}{6}$; ergo ille ad hanc $= 6 : 4 = 3 : 2$.

THEOREMA XII.

Sit circulus AZBV (Fig. 22.) genitor cycloidis SAC, erit quodlibet rectangulum ex linea compoſita ex ſemiordinata circuli, & cycloidis in interceptam in-

ter curvam circularem, & cycloidalem æquale quadrato tangentis ductæ a puncto communi ſemiordinatæ, & curvæ cycloidalis ad circulum genitorem.

Demonſtratio.

Demonſtratio patet ex elem.; nam ſi a puncto quolibet extra circulum ad circumferentiam concavam ducitur linea, rectangulum ſub tota linea, & parte intercepta inter punctum aſſumptum, & peripheriam eſt æquale quadrato tangentis ab eodem puncto ad circulum ductæ.

Corollarium I.

Patet ſuperiorem proprietatem competere etiam rectangulis ex lineis compoſitis ex ſemiordinatis circuli, & aliarum curvarum in interceptas inter utramque curvam; ut ſunt conchois Nicomedis, ſi circulus ſit deſcriptus, cuius diameter ſit axis conchoidis, parabola, & hyperbola habentes parametrum, & axem tranſverſum majorem diametro circuli, & aliæ.

Corollarium II.

Hinc, cum dictum ſit Theor. VIII. quodlibet rectangulum ZTV (Fig. 17.) parabolæ habentis parametrum æqualem diametro circuli æquale eſſe quadrato

cor-

correſpondentis abſciſſæ, erit æquale etiam quadrato tangentis ductæ ex puncto T ad circulum, & hæc erit æqualis correſpondenti abſciſſæ.

Corollarium III.

Pariter Corol. I. Theor. X. dictum fuit rectangulum ZTV (Fig. 19.) hyperbolæ æquilateræ, cuius axis tranſverſus æqualis eſt diametro circuli, æquale eſſe duplo quadrato correſpondentis abſciſſæ; ergo idem rectangulum æquale erit quadrato tangentis ex puncto T ad circulum ductæ, & quælibet tangens erit æqualis radici dupli quadrati abſciſſæ reſpondentis.

THEOREMA XIII.

Dato circulo AZBV (Fig. 22.) genitore cycloidis SAC, ſi ſupponitur diviſa ſemicircumferentia AVB in partes infinite parvas, & æquales, & intelliguntur ductæ a quovis puncto ad curvam cycloidalem lineæ VT, ut parallelæ, erit ſumma omnium harum linearum ad circulum genitorem, ut circulus genitor ad quadratum inſcriptum.

Demonſtratio.

Ex natura cycloidis quælibet VT eſt æqualis correſpondenti arcui AV circuli genitoris; ergo in noſtra

hypotheſi prima ordinata VT uni, immediate ſequens duabus infiniteſimis ſemicircumferentiæ &c. ſunt æquales; ergo huiuſmodi ordinatæ conſtituunt progreſſionem arithmeticam naturalem, & earum ſumma eſt æqualis $\frac{BC^2}{2}$; ſed area circuli genitoris, ſuppoſito eius radio æquali R, eſt æqualis $BC \times R$; ergo dicta ſumma eſt ad circulum, ut $\frac{BC^2}{2} : BC \times R$, ut $BC : 2R$, ut ſemicircumferentia ad diametrum; ſed in hac eadem ratione eſt etiam circulus genitor ad quadratum inſcriptum; ergo eſt ſupradicta ſumma ad circulum genitorem, ut circulus genitor ad quadratum inſcriptum. Q. e. d.

Corollarium I.

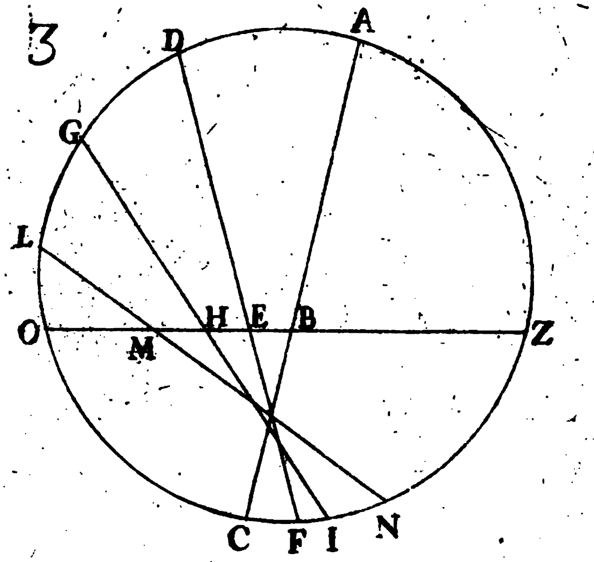
Cum ſpatium AVBCTA cycloidale ſit æquale circulo genitori, erit ſumma ordinatarum VT, ut &c. ad hoc ſpatium, ut idem ſpatium ad quadratum circulo genitori inſcriptum.

Corollarium II.

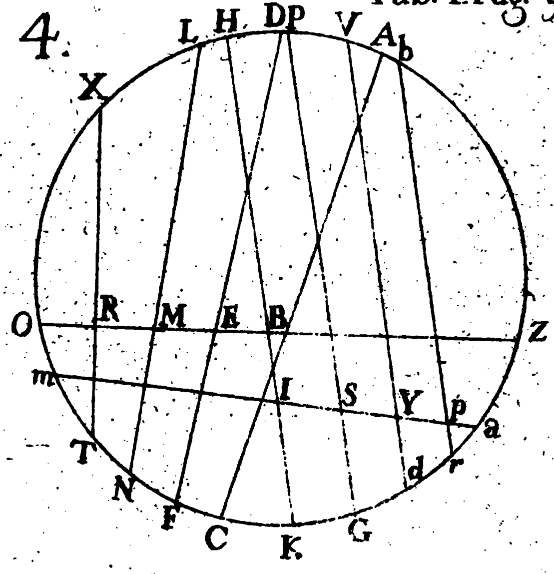
Patet ſpatium AVBCTA minus eſſe ſumma linearum VT, ut dicto modo in hoc ſpatio ductarum.

Corol-

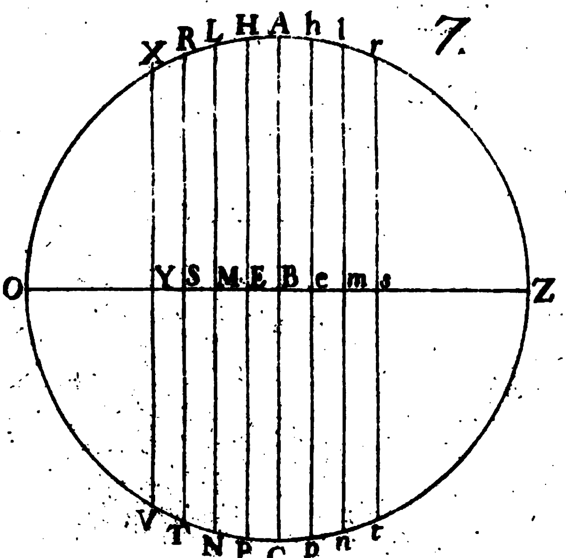
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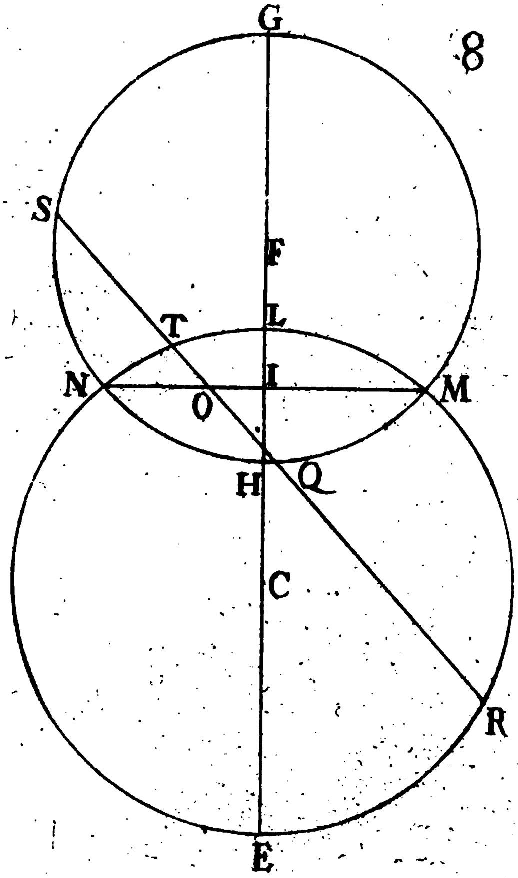
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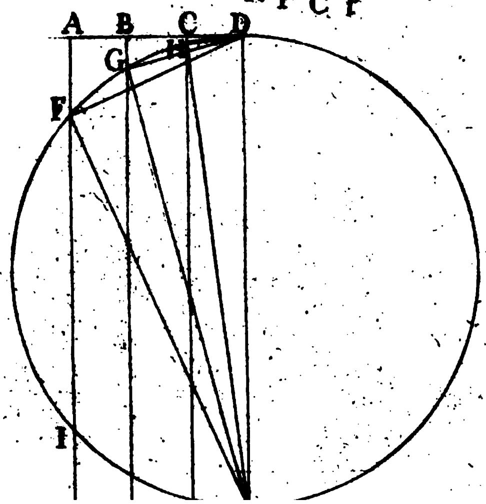


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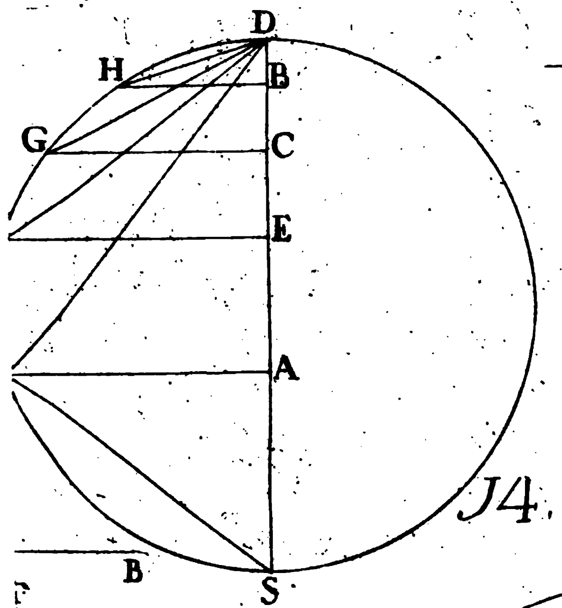
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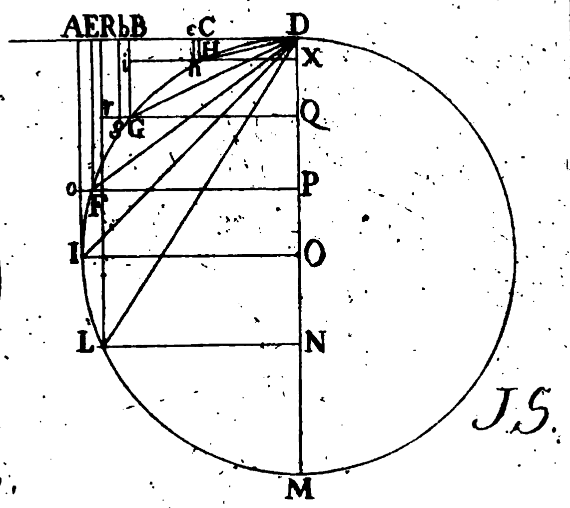


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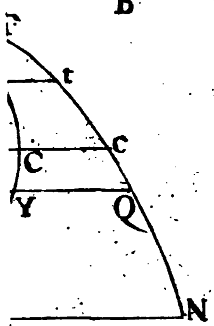
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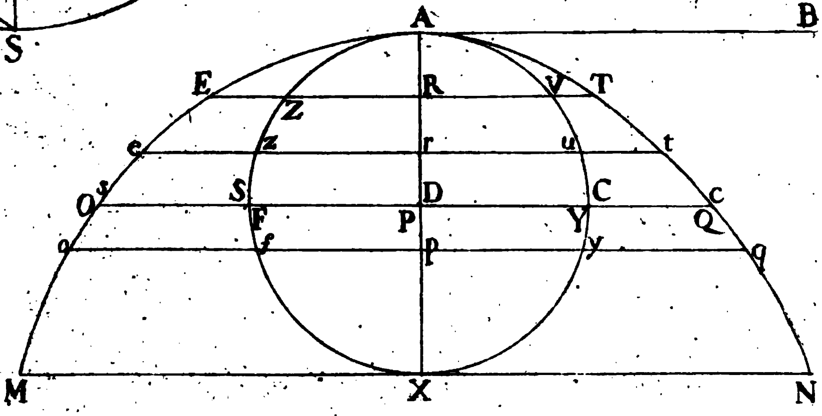

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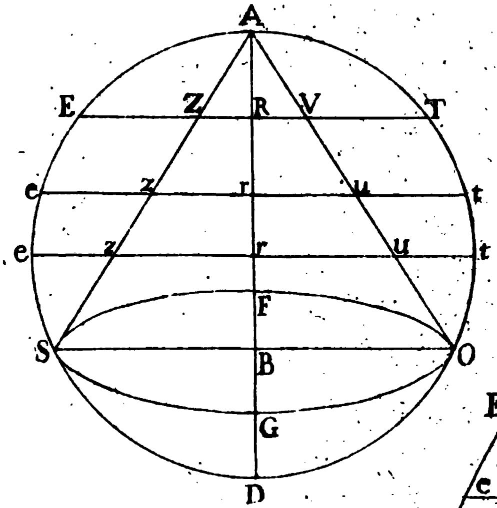

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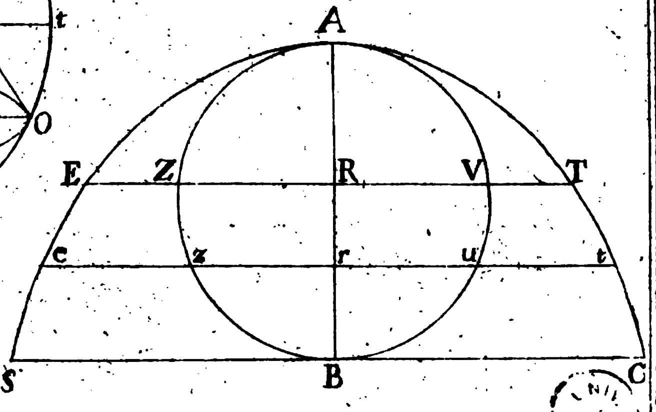

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Corollarium III.

Cum ſomma linearum TV, tu &c. in infinitum, ſit æqualis $\frac{BC^2}{2}$, ſitque $\frac{BC^2}{2}$ area trianguli iſoſcelis, & rectanguli, cuius latera æqualia adæquent ſingula ſemicircumferentiam AVB in lineam rectam productam, erit huiuſmodi triangulum ad circulum genitorem, ut circulus genitor ad quadratum ei inſcriptum; ſeu erit idem triangulum ad ſpatium cycloidale AVBCTA, ut idem ſpatium ad quadratum circulo genitori inſcriptum, vel dimidium quadrati ſemiperipheriæ AVB erit ad circulum genitorem, aut ſpatium cycloidale AVBCTA, ut circulus genitor, vel ſpatium cycloidale ad quadratum circulo inſcriptum.

THEO-

THEOREMATA

Ad Triſectionem arcus ſpectantia.

CUm ſextum volumen Hiſtoriæ, quam litterariam dicunt, quod nempe a Martio incipit Anni milleſimi ſeptingenteſimi quinquageſimi ſecundi, & uſque ad Septembrem eiuſdem anni protrahitur, fortuito legerem; incidi in geometricam ſolutionem Problematis de Enneagono circulo inſcribendo, propoſitam a Domino Joſeph Antonio Chaurard Du Clos. Problemate autem vix inſpecto efficere non potui, quin mirarer præclarum hunc Auctorem ad tam ſublime inventum perveniſſe, pro quo infelici prorſus exitu univerſum Mathematicorum genus hactenus deſudavit. At vero multo maiori correptus fui admiratione, cum, quid de huius problematis natura omnes ſentiant Mathematici, aliquanto attentius conſiderare cœpi. Veterum ſiquidem Mathematicorum, atque recentium communis opinio eſt, commemoratum problema, & generatim problema omne de Triſectione arcus, Sectionum conicarum ope, reſolvi, ut revera per has tantum curvas ea varie reſolvunt; nec deſunt, qui huiuſmodi problemata e numero problematum, quæ Plana appellantur, quæque per rectas

lineas,

lineas, & circulos expediuntur, putent eſſe eliminanda, cum ea natura ſua ſolida eſſe judicent. Verum non ita facile ipſis poſſum aſſentiri, niſi id geometricis demonſtrationibus probatum videam, quæ me hac de re evidenter perſuadeant. Si enim ab ipſis quæritur, cur hæc problemata ſolida ſint, hoc unum, ut plurimum reſpondent: quia, quomodocumque per Analyſim eorum ſolutio tentetur, ea ſemper in æquationes ſolidas incurrit. At novum non eſt apud Algebriſtas plura problemata tamquam ſolida ſe ſe offerre, quæ immutatis principiis, per quæ ſolida apparebant, iiſque ſubſtitutis, quæ ad problematis naturam proximius accedunt, e numero planorum non excluduntur. Ut igitur problema de Triſectione arcus ſolidum eſſe rite concluderetur, probandum eſſet, ea fuiſſe aſſumpta in ſolutione principia, quæ problematis naturam intimius pervadunt, eaque ad æquationem ſolidam conducere: quod cum non ita facile fieri poſſit, improbandus profecto non erit Auctor ſolutionis de Enneagono circulo inſcribendo, ſi eam tentavit ſolo circulo, & lineis rectis, quaſi ignoraret naturam problematis, quod ſolvit; imo laudandus, ſicuti commendandi eſſent quotquot aliorum dictis haud facile credentes, aliquid, caute quidem, noſtris etiam temporibus de iis ſcriberent problematibus, quæ Geometriæ legibus non eſſe ſubjecta quamplurimi gravis fortaſſe aſſerunt; fieri enim ſic poſſet, ut uni geometricæ co-

cognitioni altera accederet, donec tot obtinerentur, quot ſufficerent ad ea ſolvenda geometrice problemata, quæ ad hanc diem per Geometriam inſolubilia fuerunt iudicata. Sed quidquid de his ſit, Dominus Chautard Du Clos de Enneagono regulari circulo inſcribendo Problema, vel ſolidum illud ſit, vel planum, geometrice reſolvit. Examinanda igitur eſt eius geometrica conſtructio, eiuſque demonſtratio, quæ ſi vera erunt, inconcuſſum erit propoſitum problema, æque ac univerſale problema de Triſectione arcus, eſſe problema planum; ſi falſa, adhuc ſub lite erit, utrum plana ſint, an ſolida dicta problemata, donec evidentes hac de re ab aliquo Mathematico tradantur demonſtrationes. En quemadmodum ab Auctore ſuo proponitur,

PROBLEMA.

Dato circulo Enneagonum regulare inſcribere.

Datus circulus eſt ABC; (Fig. I.) ei Enneagonum regulare eſt inſcribendum.

Conſtructio.

I. A centro A ducatur linea ABD æqualis diametro eiuſdem circuli. Fiat triangulum æquilaterum ADE, & huic inſcribatur circulus BFH. Ducatur recta DC, quæ erit perpendicularis AE, & ſecabit circulum BFH in

in duas partes æquales per prop. 19. lib. 3. Elem.

II. Fiat centrum in A, & intervallo AF (punctum F determinatur a perpendiculari DC ſecante circumferentiam circuli BFH) deſcribatur circulus ALF.

III. Fiat centrum in C, ubi linea AE ſecat circumferentiam ABC, & deſcribatur circulus ABE æqualis circulo ABC.

IV. A centro A ad punctum Z ducatur linea AZ, punctum Z eſt punctum illud, ubi circumferentiæ duorum circulorum ALF, ABE ſe mutuo ſecant verſus F.

Dico arcum CT circuli ABC eſſe nonam partem circumferentiæ eiuſdem circuli, ac propterea eius chordam in dicto circulo novies repetitam Enneagonum exhibere, quod petebatur.

Præparatio.

V. Fiat ſector ALM, qui eſt ſexta pars circuli ALF. Producatur radius LA uſque ad circuli ALZ circumferentiam in X.

VI. Ducatur linea ZX. A puncto M ducatur linea MG ad punctum G, ubi circumferentiæ circulorum ABE, ALZ ſe mutuo ſecant verſus X: hæ duæ, cum ſint æquales per prop. 29. 3., ſe mutuo ſecabunt in eodem puncto, ubi eædem ſecant lineam AE per 26. 1. Fiat centrum in eodem puncto, facta hypotheſi illud eſſe

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punctum C, & intervallo CM, vel CX deſcribatur circulus MSX.

VII. Ducatur diameter GAP; linea PM, quæ producatur in R. Ducatur linea LZ, quæ pariter producatur in I.

Demonſtratio.

VIII. Angulus ſegmenti PML eſt æqualis angulo ſegmenti LZM per prop. 24. 3: angulus ſegmenti PML eſt minor angulo curvilineo PMS per prop. 16. eiuſdem libri: ergo angulus ſegmenti LZM erit multo minor angulo curvilineo LZA; cum hic angulus maior ſit angulo PMS: nam, ut evidens eſt, cum arcus ZBA magis ſemper recedat ab arcu ZML, quam recedat arcus MS ab arcu MLP, neceſſe eſt angulus curvilineus LZA maior ſit angulo curvilineo PMS: contra arcus ZE, cum minus ſemper recedat ab arcu ZN, quam arcus MO ab arcu MN, angulus curvilineus NZE erit minor angulo curvilineo NMO; ſed anguli mixtilinei NZI, NMR ſunt æquales per ax. 3. lib. 1.; ergo uterque eorum erit maior angulo curvilineo NZE; nam eorum uterque eſt maior angulo curvilineo NMO per 16. 3; ergo linea LZI eſt tangens circuli ABE ad punctum Z per def. 2. 3: ſed angulus LZX eſt rectus per prop. 31. 3; ergo linea ZX tranſit per centrum C circuli ABE per prop. 19. 3; quare AZC eſt iſoſceles

per

per def. 5. 1. Et ſimiliter triangulum AZX eſt iſoſceles per eandem definitionem; ergo angulus AXZ erit æqualis angulo ZAC per prop. 5. 1; ergo arcus LMZ erit duplus arcus ZN per prop. 20. 3; & arcus ZN erit tertia pars arcus LN: arcus etiam CT erit tertia pars arcus CK, quia hi duo circuli ſunt concentrici: ſed arcus LN eſt tertia pars circumferentiæ circuli ALF, quemadmodum eſt arcus CK tertia pars circumferentiæ circuli ABC ex num. 5; ergo arcus CT eſt nona pars circumferentiæ eiuſdem circuli ABC; ac proinde repetatur novies eius chorda in circulo ABC, & habebitur Enneagonum, quod quæritur. Q. e. d.

Hæc igitur ſuperius adducta Enneagoni regularis conſtructio, & demonſtratio ea eſt, quam Mathematicis propoſuit Dominus Chautard Du Clos. Solutio autem problematis tamdiu deſiderata, ſuoque ab Auctore tradita, nullo præter conſuetudinem verborum præmiſſo apparatu, quæ eiuſdem excellentiam, novitatem, utilitatem, difficultatemque ſignificarent, me profecto de eius modeſtia certiorem facit, mihique apertiſſime declarat, quidnam eum impulerit, cum Mathematicorum cœtui ſuum communicavit inventum: ſolus nimirum veritatis amor, ac promovendæ Matheſeos deſiderium. Quæ duo, cum in me etiam plurimum poſſint, hinc potero, citra Auctoris, quem ſummopere veneror, offenſam, quid ſentiam tum de conſtructione, tum de

demonſtratione, ne minus periti in mathematicis facultatibus fallantur, ſincere aperire.

Demonſtratio ergo Auctoris non innititur principiis apud omnes Mathematicos certis; ſupponit enim angulum contactus eſſe quantum, repugnantibus Peletario, Galilæo, Valliſio, Borellio, aliiſque magnæ notæ Mathematicis; nec ſane tam facile ipſi erit id, quod aſſumit evidenter demonſtrare. Quod ſi cum laudatis Viris ponamus angulum contactus eſſe nullum, & angulos omnes ſemicirculorum eſſe æquales, tunc demonſtratio longe aliter concludet; nam primo non amplius verum erit angulum ſegmenti PML minorem eſſe curvilineo PMS, quia angulus curvilineus PMS excedit angulum ſegmenti PML angulo contactus PMS; ſed angulus contactus eſt nullus; ergo angulus ſegmenti PML = PMS curvilineo. Secundo vel linea ZX tranſit per centrum circuli AZE, quod illud eſt, quod eſt demonſtrandum, vel non: ſi primum, cum angulus XZL ſit æqualis GMP per conſt., LZM = PML, erit angulus mixtilineus XZM = GML mixtilineo, & detractis utrinque angulis ſemicirculi æqualibus XZA, GMS, erit curvilineus AZP = SMP curvilineo: pariter cum angulus ſemicirculi XZE ſit æqualis GMO, angulus ſegmenti XZN = GMN, his ſubtractis, erit angulus NZE = NMO; cumque angulus NMO ſit æqualis NMR, quia angulus contactus eſt nullus; ergo anguli NMR,

NMO,

NMO, NZI, NZE ſunt inter ſe æquales. Si ſecundum, hoc eſt ſi linea ZX non tranſiret per centrum circuli AZE, tunc unus angulus ſegmenti, puta XZE, maior eſt angulo ſemicirculi, alius XZA eodem minor: cum ergo angulus XZL ſit æqualis GMP, angulus ſegmenti XZA per hyp. ſit minor angulo ſemicirculi, ac proinde minor angulo GMS, his detractis ab æqualibus XZL, GMP, erit mixtilineus AZL maior mixtilineo SMP; & additis angulis ſegmenti æqualibus LZM, PML, erit angulus curvilineus AZP maior curvilineo SMP; ſimiliter cum angulus XZI ſit æqualis GMR, angulus XZN ſegmenti ſit æqualis angulo ſegmenti GMN, his ſubductis, erit mixtilineus NZI = NMR mixtilineo; ſed NMR = NMO curvilineo, quia angulus contactus eſt nullus; ergo mixtilineus NZI = NMO; ſed quia angulus ſegmenti XZE per hyp. maior angulo ſemicirculi GMO; ergo, demptis utrinque angulis ſegmenti æqualibus XZN, GMN, curvilineus NZE eſt maior NMO curvilineo; ergo NZE eſt maior NZI mixtilineo.

Sed his omiſſis, quæ omnino ſunt abſtracta, & dubia, clarius procedo, & dico demonſtrationem non ſubſiſtere defectu conſtructionis. Quod, ut appareat, ſit deſcripta Fig. 2. eo modo, quo ab Auctore conſtruitur pro ſolutione Enneagoni, erit AZ ea linea, quæ per ipſum ſecat arcum CT nonam partem circumferentiæ

cir-

circuli ABC. Ducatur autem AF a centro A ad punctum F, ubi circulus BFH ſecat perpendicularem CD, & dicatur diameter $AE = AD = 2a$: erit $AB = AC = BD = a$; &, ob triangulum ADC rectangulum, erit $AD^2 = DC^2 + AC^2$, hoc eſt $4a^2 = a^2 + DC^2$, adeoque $DC^2 = 3a^2$, & $DC = a\sqrt{3}$. Quia vero DB tangit circulum BFH in B, erit $BD^2 = DC \times DF$, hoc eſt $a^2 = a\sqrt{3} \times DF$, ac proinde $DF = \frac{a}{\sqrt{3}}$; erit ergo $CD - DF = CF = a\sqrt{3} - \frac{a}{\sqrt{3}} = \frac{2a}{\sqrt{3}}$; ſed $AF^2 = AC^2 + CF^2 = a^2 + \frac{4a^2}{3} = \frac{7a^2}{3}$; ergo AF^2, ſeu $AZ^2 = \frac{7a^2}{3}$, hoc eſt quadratum chordæ AZ, quod per Auctorem debet eſſe æquale quadrato chordæ 100°, ut eius conſtructio ſit accurata, habetur per hanc expreſſionem $\frac{7a^2}{3}$; ergo chorda $AZ = a\sqrt{\frac{7}{3}}$, & $\frac{AZ}{2}$, quæ quantitas ad mentem Auctoris debet eſſe ſinus rectus arcus 50° eſt æqualis $\frac{a}{2}\sqrt{\frac{7}{3}}$. Supponatur igitur radius $AC = a$ diviſus in 10000000 partium æqualium, quemadmodum ſupponi ſolet diviſus in Trigonometria ad inveniendos ſinus, atque tangentes; erit $AZ^2 = \frac{7a^2}{3} = \frac{700000000000000}{3} = 233333333333333\frac{1}{3}$,

& ex-

& extracta radice quadrata erit $a\sqrt{\frac{7}{3}} = 1527525$ quam proxime, & $\frac{a}{2}\sqrt{\frac{7}{3}} = 7637626$. At, ſi tabulas ſinuum conſulamus, inveniemus ſupradictum ſinum eſſe quam proxime ſinum rectum arcus 49° 47′ 49″; non autem 50°. Chorda igitur AZ eſt minor chorda 100°; ergo chorda ZE eſt maior chorda 80°; ergo arcus ZE eſt maior 80°; ſed hic eſt duplus arcus CT; ergo CT eſt maior 40°; ergo eſt maior nona circumferentiæ parte. Patet itaque conſtructionem Problematis deficere; adeoque in Demonſtratione paralogiſmos latere.

Verum contra ea, quæ dixi, inſurgere quis poſſet, mihique opponere fundamenta adducta evidenter non concludere minime accuratam eſſe conſtructionem Problematis de Enneagono circulo inſcribendo, cum ea dependeant a tabulis trigonometricis, de quarum veritate dubitari fortaſſe poſſet. Sed iſta obiicienti ſufficiat modo communis Mathematicorum conſenſus, qui eas veluti exactas receperunt, donec aliquo præmiſſo theoremate eiuſdem defectum ex alio capite mihi liceat inferre. Antequam vero aliquod proponam Theorema, placet quædam præmonere. Univerſum hoc Opus, quod ſcribo, quomodocumque vel ad Triſectionem arcus conducere poſſit, vel ad errores evitandos, qui ab iis committi ſolent, qui triſectiones tentant, quod ignoro, improbo

probo ſtudio, ac diuturno duorum annorum decurſu ſucciſivis horis compoſui. Si aſſiduis laboribus inventorum meorum fructus reſpondiſſent, utiliſſimum profecto, atque doctrina plenum Mathematicis opus exhiberem; ſed non hæc tentanda ſunt Problemata, quæ Mathematicis illudunt, ut ſtudia noſtra feliciter cadant. Nonnullis igitur Theorematibus, & aliquo Problemate totum opus perſtringam, deducamque quidquid ex iis deduci poſſe cognovi. Fateor autem in theorematis conficiendis, rebuſque adnotandis, quæ ex iis conſequebantur, non earum uſum, & utilitatem, ſed potius veritatem conſideraſſe; nec ſane immerito, cum de rebus agatur, de quarum natura vel parum, vel nihil intelligitur: Etenim, ſi a Matheſi quovis tempore eæ omnes arcendæ fuiſſent propoſitiones, quarum uſus, ac utilitas non attingebatur, nunquam innumera propemodum præclara, atque utiliſſima inventa in lucem prodiiſſent. Neglexiſſet fortaſſe Apollonius Sectionum conicarum doctrinam, Proiectorum vias non demonſtraſſet Galilæus, Planetarum tramites non cognoviſſet Keplerus, pythagoricum inventum, vel propoſitionem 36. lib. 3. non ſcripſiſſet Euclides, Operum, quæ libella fiunt corrigendorum modum ignoraſſent Recentiores, tacuiſſent Veteres, ut alia omittam, Progreſſionum geometricarum, & arithmeticarum comparationem, Logarithmorum doctrina tam utilis in tenebris denique latuiſſet.

tuiſſet. Quidquid ergo in hiſce meis Scriptis verum, atque evidenter demonſtratum continebitur, ſpero Mathematicos æquo animo eſſe excepturos, etſi nullius momenti inventa, eorumque attentione indigna apparere, poſſint, quia ſæpe, quod ineptum primo intuitu cenſetur, commodum ſucceſſu temporis evadit. Quod ſi hæc mea Scripta eorum etiam approbationem obtinebunt, ſciant, quotquot hæc legunt, laudem omnem D. Joſeph Antonio Chautard Du Clos eſſe tribuendam, qui, licet in ſua Problematis ſolutione aliquid erraverit, me tamen principiis ſuis ad talia excitavit, ad quæ aliter nunquam manum admoviſſem.

THEOREMA I.

Si duo circuli KBC, AZE (Fig. 3. n. 1. 2. 3. 4.) æquales ita ſint deſcripti, ut circumferentia unius tranſeat per centrum alterius, & centro A radio quocumque AZ deſcribatur circulus LZG ſecans circulum AZE in duobus quibuſcumque punctis Z, & G; Dico, quod ductis a ſectionibus Q, & G per centrum A rectis LX, GP, & a puncto G ad L linea GL ſecante peripheriam AZE in O, ſi addantur QO, & ZO, hæ lineæ erunt ſemper inter ſe æquales, & ducta QZ triangulum QOZ iſoſceles.

Demonſtratio.

Triangulum AGL (Fig. 3. n. 1. 2. 3.) eſt iſoſceles per conſtructionem; ergo AGO = AQO = AZO = ALO; ergo QOL eſt triangulum iſoſceles, & QO = LO: ſed ALZ eſt triangulum iſoſceles per conſtructionem; ergo angulus ALZ = AZL; ſed ALO = AZO; ergo, his detractis (in Fig. 3. n. 1. 2.), iiſque additis (in Fig. 3. n. 3.), OLZ = OZL; ergo LO = OZ = QO; ergo triangulum QOZ iſoſceles. In Figura autem 3. n. 4., quando nempe radius AZ = AB, patet triangulum QOZ eſſe iſoſceles; nam tunc puncta B, & Z concurrunt in unum, item Q, & G, ſimiliter A, & O; adeoque AB, AQ, utpote duo radii, ſunt æquales. Quod erat demonſtrandum.

Corollarium I.

Cum triangulum QOZ ſit iſoſceles, ducta linea OE, erit per Corol. V. prop. 19. partis 1. Inſtit. geometricarum P. Ab. Grandi, $QR \times RZ + OR^2 = ZO^2$; ſed $QR \times RZ = OR \times RE$; ergo $OR \times RE + OR^2 = ZO^2$; ſed $OR \times RE + OR^2 = EO \times OR$; ergo $EO \times OR = ZO^2$, & $EO : ZO = ZO : OR$. Cum autem eodem modo demonſtrari poſſit, ducta linea OG, rectangulum $GO \times OF = ZO^2$, hinc patet, quod ſi cir-

circulo aliquod erit inſcriptum triangulum iſoſceles QOZ, & a vertice O ad baſim QZ erunt ductæ quæcumque rectæ omnes protractæ uſque ad circumferentiam in E, & G, rectangula omnia ex totis EO, GO in partes OR, OF erunt inter ſe æqualia, & æqualia quadrato ZO lateris ejuſdem trianguli. Si producatur linea ZQ (Fig. 3. n. 1.) extra circulum in q, & ducatur Oq ſecans circulum OQE in g, etiam rectangulum qOg erit æquale quadrato QO; nam rectangulum ZqQ + QO^2 = qO^2 = qOg + Oqg; ſed Oqg = ZqQ; ergo, his demptis, qOg = QO^2.

Corollarium II.

A puncto Z ad G, X, P, L (Fig. 3. n. 1.) ducantur lineæ ZG, ZX, ZP, ZL: Cum ſemicircumferentia AZE ſit æqualis AGE, arcus AZ = AG erit arcus ZE = GE; linea ergo AE, quæ tranſit per centrum C, & ſecat bifariam arcum ZEG, ſecat bifariam, & ad angulos rectos chordam ZG. Eſt igitur AE parallela PZ, & arcus Ap = ZE. Similiter, quia angulus ZLG = ZPG = $\frac{1}{2}$ GAZ = EAZ = EOZ = OZL; ergo OE eſt parallela LZ, & arcus Oy = ZE = Ap. Quia vero angulus GZX = GLX = AGO, arcus GI = AO; ſed QG duplus AO; ergo GI = QI. Quod ſi a puncto G per communem ſectionem H linearum AE, & ZX ducatur GM, erit etiam arcus Zt = GI = Bt.

Idem fere eodem modo demonſtratur in reliquis figuris n. 2. 3. 4.

Corollarium III.

Cum ſit arcus MZ (Fig. 3. n. 1. 2. 3. 4.) interceptus inter lineas GZ, & GM ſimilis arcui Zt, & arcui BT, patet ducta linea AM, hanc tranſire per communem ſectionem B circulorum æqualium.

Corollarium IV.

Quia totus arcus BE (Fig. 3. n. 1.) eſt duplus AB, & arcus BZ, ſeu QG duplus AO; ergo eſt ZE duplus OB, adeoque $OB = By$. Eſt ergo arcus ty ſemidifferentia inter arcum BZ, & arcum ZE; nam eſt arcus $BZ = \frac{1}{2} BZE$, ſi huic addatur ſemidifferentia inter arcum BZ, & ZE; ſed eſt $\frac{1}{2} BZE = By + Zt$; ergo, cum $By + Zt + ty = BZ$, eſt ty ſemidifferentia inter BZ, & ZE. Pariter eſt pB ſemidifferentia inter arcum py, & arcum Op; nam dimidium ſummæ arcuum yp + pO eſt By; ſed By + Bp efficit arcum maiorem py, & dimidium ſummæ OB — pB dat arcum minorem Op; ergo pB eſt ſemidifferentia. Quia autem eſt arcus $Ap = Oy$, dempto arcu Op, eſt $AO = py = Bt$; adeoque $pB = yt$. Sunt igitur in proportione arithmetica arcus $BZ : ZE = py : Op$, & $BZ + Op =$ ZE

ZE + py. Idem demonſtratur in Figura 3. n. 2.; nam arcus BE eſt duplus AB, & BZ, ſeu QG eſt duplus AO; ergo ZE eſt duplus OB; ſed etiam BZ eſt duplus Bt; ergo OB + Bt = $\frac{1}{2}$ BZE; ſed OB + Bt + ty = ZE; ergo ty eſt ſemidifferentia inter arcum ZE, & BZ. Pariter eſt Bp ſemidifferentia inter arcum Op, & py; nam dimidium ſummæ arcuum Op + py = OB, ſeu By; ergo, cum OB + Bp ſit quantitas maior Op, & By — Bp quantitas minor py, eſt pB ſemidifferentia. Eſt autem pB = ty; nam Ap = ZE; ſed ZE = Oy; ergo Ap = Oy, &, dempto Op, AO = py; ſed AO = Bt; ergo Bt = py, &, addito tp, Bp = ty. Eſt igitur in proportione arithmetica ZE : BZ = Op : py. Hæc eadem vera ſunt etiam in Figura 3. n. 4., ut facile patet. In Figura autem 3. n. 3., in qua nempe ZB eſt pars arcus ZE, erit ty ſemiſumma arcuum ZE + ZB; nam, quia arcus BE eſt duplus AB, & arcus ZB duplus AO, erit ZE duplus OB; ſed ZE = OBy, quia OZ = Ey, &, addito Zy, Oby = ZE; ergo ZE duplus By; ſed ZB eſt duplus tB; ergo tB + By = $\frac{ZE + ZB}{2}$. Ex dictis conſtat etiam in Figura 3. n. 1. 2. 3. arcus By, & Zt, qui ſimul efficiunt ſemiſummam arcus BZE eſſe ſimul 60°, & in Figura 3. n. 4. ſemidifferentiam arcus ZE ab arcu BZ eſſe 60°.

Corol-

Corollarium V.

Ex Corollario præcedenti patet igitur portionem arcus ty (Fig. 3. n. 1.) interceptam inter lineam LZ, & lineam GM eſſe ſemidifferentiam inter arcum BZ, & arcum ZE. Item clarum eſt portionem arcus inter communem ſectionem B circulorum KBC, AZE, & ſectionem y per lineam LZ eſſe dimidium arcus ZE. Pariter manifeſtum eſt arcum Bt, vel Zt eſſe dimidium arcus BZ.

Corollarium VI.

Si ergo linea ducta a puncto L ad punctum Z iuxta propoſitam conſtructionem circulum tangeret in puncto Z, tunc puncta Z, & y (Fig. 3. n. 1.) concurrerent in unum punctum; quare eſſet arcus ZE duplus BZ; adeoque arcus BE eſſet triſectus; ac proinde etiam arcus BC, hoc eſt circulus KBC eſſet diviſus in novem partes æquales.

Corollarium VII.

Cum linea OE (Fig. 3. n. 1. 2. 3. 4.) ſit parallela lineæ LZ, & linea ZX ſit perpendicularis LZ, quia angulus LZX in ſemicirculo, eadem ZX erit perpendicularis OE; erit ergo triangulum RZV iſoſceles: nam in

In duobus triangulis R Z S, V Z S angulus R Z S = V Z S, quia arcus QI = GI; anguli R S Z, V S Z recti, latus Z S commune; adeoque Z R = Z V. Erunt igitur ſimilia etiam duo triangula O R Z, E V Z; nam angulus O R Z = E V Z, quia Z R S = Z V S; E O Z = G Z E, quia arcus Z E = G E; ergo O R : R Z = Z V : V E; ſed Z R = Z V; ergo O R : Z V = Z V : V E.

Corollarium VIII.

Si linea O R (Fig. 3. n. 1.) evaderet æqualis lineæ Z R, cum ſit O R : R Z = Z V : V E; tunc lineæ O R, R Z, Z V, V E eſſent æquales; adeoque triangula O R Z, Z V E eſſent æquiangula, æqualia, & iſoſcelia; ac proinde arcus G E = Z E = Z O = Q O.

Corollarium IX.

Eſſet ergo triangulum O Z E (Fig. 3. n. 1.) iſoſceles; quare angulus O E Z = E O Z = O Z L. Eſſet igitur L Z tangens circuli A Z E in Z; nam angulus O Z L = O E Z in ſegmento alterno: Eſſet arcus Z E duplus B Z per Corollarium VI. Linea Z X, quæ perpendicularis eſſet tangenti L Z tranſiret per centrum C circuli A Z E. Uno verbo Figura hæc in Figuram 4. abiret, in qua linea A Z ſecaret arcum C T, qui eſſet nona pars peripheriæ circuli K T G.

Corol-

Corollarium X.

Suppoſita ergo Figura 4. eo modo conſtructa, quo dixi, ita nempe, ut exhibeat ſolutionem problematis de Enneagono circulo inſcribendo, perutile erit tum ad ſynthesim, tum ad analyſim promovendam omnes linearum relationes, omniaque ſymptomata examinare, quæ in dicta triſectione occurrerent. Quare, cum primo arcus ZE ſit duplus BZ per Corol. VI., ſitque totus arcus BE 120°, erit ZE 80°, BZ 40°. Secundo, cum ſit arcus ZE 80°, erit AZ arcus 100°; ſed arcus ZO = ZE; ergo arcus AO erit 20°. Tertio, cum ſit GE = ZE = ZO per Corol. VIII., erit arcus GZO 240°, & erit triſectus; ergo arcus GO erit 120°, adeoque latus GO erit latus trianguli æquilateri. Quarto, cum per Corol. II. debeat eſſe arcus Ap = ZE, erit Ap = ZO, &, dempto Op, erit arcus AO = Zp; ſed BZ duplus AO ex dictis; ergo BZ duplus Zp; ſed BZ duplus Zt per Corol. II.; ergo puncta t, & p linearum GM, PZ concurrunt in unum.

Corollarium XI.

Cum circulus geometrice dividi poſſit in decem partes æquales, ſi iuxta ſuperiorem conſtructionem dividi etiam poſſet in novem partes æquales, poſſet etiam geo-

geometrice dividi in nonaginta partes æquales, & quemlibet arcum biſecando in centum octoginta partes, & iterum quemlibet arcum bifariam dividendo in tercentum ſexaginta partes æquales; ergo circumferentia circuli dividi poſſet geometrice in omnes gradus, in quos ſolet a Geometris dividi; ergo circulo inſcribi, & circumſcribi poſſet quælibet figura, cuius latera ſubtenderent arcus ex integris gradibus conſtantes, vel conſtantes ex integris gradibus, & dimidiis, quartis, octavis partibus gradus &c.

Corollarium XII.

Quia per Corol. VIII. arcus GE = ZO erit linea GL parallela ZE; & AZ, quæ eſt perpendicularis ZE, etiam parallelæ GL erit perpendicularis; & quia LO eſt parallela ZE, & LZ parallela OE per Corol. II., erit OLZE parallelogrammum. Quia, vero LZ eſt tangens circuli AZE, erit angulus in ſegmento alterno OGZ = OZL = ZOE = OEZ = OLZ = OZR; adeoque triangula LZG, LOZ, OZR erunt iſoſcelia, & ſimilia. Erit igitur GL : LZ = LZ : ZO = ZO : OR, hoc eſt, lineæ LZ, & ZO erunt duæ mediæ proportionales inter GL, & OR.

Corollarium XIII.

Si in puncto E erigatur tangens Eω, & producatur LZ, donec concurrat cum Eω in ω, etiam triangulum EZω eſt ſimile, & æquale triangulo VZE, & RZO; ergo eſt etiam GL : ZG = ZG : ZE = ZE : Zω, hoc eſt linea GL compoſita ex latere trianguli æquilateri inſcripti circulo AZE, & ex chorda octoginta graduum eiuſdem circuli, chorda ZG centum ſexaginta graduum, chorda ZE octoginta graduum, tangens Zω quadragima graduum ſunt in proportione geometrica continua.

Corollarium XIV.

Linea AZ (Fig. 5.) eſt perpendicularis GL, ut demonſtravi; lineæ PL, & GX ſunt perpendiculares GL, quia anguli PLG, XGL ſunt in ſemicirculo; ergo lineæ PL, AZ, GX ſunt inter ſe parallelæ: ſed hoc dato angulus AZX = AXZ = ZXG; ergo, quia in triangulis QXI, GXI anguli IXG, IXQ ſunt æquales, anguli QIX, GIX ſunt recti, latus XI commune; erunt latera QX, GX æqualia, & triangulum QXG iſoſceles; ſed QX = TZ; ergo GX = TZ = PL.

Corol-

Corollarium XV.

Si ergo a puncto P per punctum T ducitur linea PTH, hæc eſt parallela lineæ LZ, lineæ QG; cumque triangulum QXG ſit iſoſceles, & ſimile triangulo KAT, linea PT tranſibit per punctum K, ubi linea AK æqualis radio AT; quare triangula QXG, PLK, TZN erunt iſoſcelia, & æqualia.

Corollarium XVI.

Quia ωLZH eſt parallelogrammum, angulus ωLZ = ωHZ = ZPL. Cum igitur in triangulis PZH, PZL, RZL angulus PHZ ſit æqualis ZPL = RLZ, angulus ZPH = PZL, triangula erunt ſimilia; ergo erit PH : PZ = PZ : LZ = LZ : RZ, hoc eſt erunt lineæ PZ, LZ duæ mediæ proportionales inter duas PH, & RZ.

Corollarium XVII.

Cum linea ZH ſit circuli LZG tangens, patet eſſe $ZH^2 = PH \times HN$. Et cum per Coroll. V. prop. 19. Inſtit. Geom. P. Grandi, ZH^2 ſit æquale $TH \times HN + ZN^2$; erit ergo $PT \times HN + TH \times HN = TH \times HN + ZN^2$; &, dempto utrinque rectangulo $TH \times HN$, erit $PT \times HN = ZN^2$; ac proinde $PT : ZN = ZN : HN$.

Corollarium XVIII.

Ducta a puncto P ad X linea PX, & a punctis T ad X, Z ad G lineis TX, ZG, erit parallelogrammum PLGX ſupra baſim trianguli PTX æquale parallelogrammis ſuper eiuſdem latera deſcriptis; nam PsSL = PTZL, & XsSG = XTZG.

Corollarium XIX.

Quod ſi producatur linea XG in x, & g, donec concurrat cum lineis PT, LZ productis, erit etiam PLgx parallelogrammum æquale PLGX, duplum PLZT; adeoque PLZT = TZgx = TZGX.

Corollarium XX.

Cum rectangulum AXQ = mXG, & QX = GX; erit Gm = AQ; ſed tota Gx = QK; ergo Gm = mx; ergo rectangulum M x D = DKM = Gxm = $\frac{AE^2}{2}$.

Cum igitur rectangulum MxD ſit æquale dimidio quadrato diametri, ſi a puncto x ad circulum AZEG ducatur tangens, hæc erit æqualis lateri quadrati circulo AZEG inſcribendi.

Corol-

Corollarium XXI.

Si linea ZN producatur uſque ad peripheriam circuli AZE, hæc eidem occurrit in m, ubi linea Gx eandem ſecat; arcus enim AQGm eſt menſura anguli AZN. Parallelogrammum autem AZmX eſt Rhombus.

Corollarium XXII.

Producantur AE, PD (Fig. 6.) donec concurrant in o. Ducatur a puncto Z ad punctum n interſectionis linearum Gg, Ao linea Zn; triangulum AGn erit iſoſceles; ergo erit AG = Gn; ſed AG = Xm; ergo Xm = Gn; ergo GX = mn. Cum igitur in triangulis AGX, Zmn ſit AG = Zm, GX = mn, angulus XGA = GAZ = Zmn; ergo Zn = Zm = AG = AX, & triangulum mZn = XAG.

Corollarium XXIII.

Ducatur linea QT, hæc erit perpendicularis PD; adeoque erit parallela ZX; erunt ergo ſimilia triangula AnZ, ACZ, AqT; ergo erit An : AZ = AZ : AC = AC, ſeu AT : Aq; erunt igitur AZ, AT duæ mediæ proportionales inter duas An, ſeu PZ, & Aq.

Corol-

Corollarium XXIV.

Quia PZ eſt parallela An, triangulum PHZ eſt ſimile triangulo HEo; ergo PH : ZH = Ho : HE; ſed PH : ZH = ZH : NH; ergo ZH : NH = Ho : HE; ergo NH × Ho = ZH × HE = MH × HD; ergo MH : Ho = NH : HD.

Corollarium XXV.

Si producatur a puncto m per T linea mT; quia mn æqualis, & parallela ZT, erit mT parallela Zn, parallela AG; ergo mT producta tranſibit per e, ubi arcus Ae eſt 60°. Tranſibit eadem mT etiam per punctum c, ubi circulus LZG ſecat diametrum AE, quia triangulum ATc eſt æquale, & æquilaterum triangulo AQT; adeoque QT = Ac = AZ.

Corollarium XXVI.

Erit ergo em chorda arcus 140°. Chorda igitur arcus 140° circuli AZE æqualis eſt ſummæ chordarum AZ 100°, & Te 20° eiuſdem circuli AZE.

Corollarium XXVII.

Cum AP ſit parallela Tc, eſt AP : Tc = Ao : co = Zn : Tc = An : Ac = Ac : AC = AC : Aq; ergo

ergo erit $Ao \times Ac = co \times An$; $Ao \times AC = co \times Ac$; $Ao \times Aq = co \times AC$; &, demptis ſecundis rectangulis a primis, $Ao \times Cc = co \times cn$, atque, ſubtractis tertiis rectangulis a ſecundis, $Ao \times qC = co \times Cc$; ac proinde $Ao \times Cc : Ao \times qC = co \times cn : co \times Cc$, hoc eſt $cn : Cc = Cc : qC$.

Corollarium XXVIII.

Quia angulus $AQy = OGZ$, arcus $Ay = OZ$, detracto Oy, $AO = Zy$; ſed chorda arcus $Zy = qC$, quia $qyZC$ parallelogrammum; ergo chorda $Zy = AO = AF = qC = En$. Eſt autem etiam $Cc = TZ = cm = mn$; ergo, quia rectangulum $Ao \times qC = co \times Cc$; erit etiam rectangulum $oAF = ocm$; & quia $cn : Cc = Cc : qC$, erit $cn : mn = mn : nE$.

Corollarium XXIX.

Ducta GN, quia hæc parallela QT, eſt $TV : VN = qC : Cz$; ſed $TV = VN$; ergo $qC = Cz$; eſt ergo zc differentia inter chordam arcus $20°$ circuli LZG, & chordam $20°$ circuli AZE.

Corollarium XXX.

Si linea PZ (Fig. 7.) evadet tangens circuli AZE, tunc puncta Z, & p concurrent in unum. Cumque AE ſit

ſit parallela PZ, eſt arcus $AZ = ZE$; ſed $AZ = AG$, & $ZE = GE$; ergo Figura GEZA eſt quadratum. Linea ZG, quæ eſt perpendicularis tangenti PZ in puncto contactus Z, tranſit per centrum C circuli AZE; item linea ZG, cum ſit perpendicularis etiam lineæ AE in puncto C, eſt tangens circuli KBC; eſt ergo $DZ : ZC = ZC : ZT$. Quare ſi dicatur radius $ZC = 1$, eſt $DZ = 1 + \sqrt{2}$, adeoque $ZT = \sqrt{2} - 1 = \frac{1}{\sqrt{2} + 1}$.

Corollarium XXXI.

Demonſtratum fuit Corol. IV. arcum Bp eſſe æqualem ty; ergo, dempto tp, ſeu tZ, $Bt = py$; ſed per Corol. II. $Bt = tZ$, ſeu tp; ergo $Bt = Zt = Zy$, ſeu py; Eſt igitur By 45° triſectus.

Corollarium XXXII.

Patet arcum PZ eſſe duplum ZN, & arcum ωT duplum TC; adeoque arcum PZN, & ωTC 135° eſſe triſectum. Cum ſit PZ diametro AE æqualis, conſtat PZ eſſe latus quadrati circulo KBC circumſcribendi, & eundem circulum KBC tangere. Manifeſtum eſt etiam triangula PZG, PAZ, AZC eſſe iſoſcelia, & ſimilia; & eſſe $PG : PZ = PZ : AZ = AZ : AC$; Quare, ſuppoſito radio = 1, erunt, ut hi numeri $2\sqrt{2} : 2 : \sqrt{2} : 1$.

Corol-

Corollarium XXXIII.

Etiam triangula AZE, OZS ſunt iſoſcelia, & ſimilia; ergo eſt $AE : AZ = ZO : OS$. Eſt igitur diameter ad chordam 90°, ut chorda 75° ad lineam OS, ſeu eſt ſinus totus ad ſinum 45°, ut ſinus 37° 30′ ad $\frac{OS}{2}$.

Corollarium XXXIV.

Per Corollarium I. eſt $AZ^2 = IZ \times ZH$; item $ZO^2 = EO \times OR$; ergo erit $IZ : AZ = AZ : ZH$, hoc eſt chorda 165° ad chordam 90°, ut chorda 90° ad lineam ZH, ſeu ſinus rectus 82° 30′ ad ſinum rectum 45°, ut idem ſinus ad $\frac{ZH}{2}$: pariter erit $EO : OZ = OZ : OR$, ideſt chorda 165° ad chordam 75°, ut eadem chorda ad lineam OR, ſeu ſinus rectus 82° 30′ ad ſinum rectum 37° 30′, ut idem ſinus ad $\frac{OR}{2}$.

Nota autem linea $OS = ZS$ per Coroll. præc., & linea OR per hoc Coroll., erit nota RS, adeoque etiam ZR, item VE per Coroll. 7.

THEOREMA II.

Si duo circuli æquales KTC, AZE (Fig. 8. n. 1.) ita ſint deſcripti, ut circumferentia unius tranſeat per centrum alterius, & centro A radio quocumque deſcribatur circulus concentricus circulo KTC, & ſecans circulum AZE in duobus quibuſcumque punctis Z, & G; dico, quod, ducta ab interſectione Q linea QA utrinque terminata ad circumferentiam circuli LHG in X, & L, & ab interſectione G per centrum linea GAP, additaque linea LH a puncto L ad H, ubi diameter AE ſecat peripheriam circuli LMG, & linea PI = LH, erit eadem PI parallela QG, & tranſibit per punctum K.

Demonſtratio.

Ducatur a centro A ad I linea AI; quia LH = PI per hypotheſim, triangulum LAH eſt æquale, & æquiangulum PAI; & quia arcus HX per conſtructionem eſt ſimilis QC, ſimilis AQ; ergo angulus XLH = QGA; ſed XLH = GPI; ergo QGA = GPI; ergo QG parallela PI; Q. e. p.m Sed, quia in triangulis QGA, PAK angulus QGA = APK, QAG = PAK, AG = AP, erit AQ = AK; ſed AQ eſt radius; ergo etiam AK eſt radius; ergo linea PI tranſit per punctum K; Q. e. ſ.n

Idem

Idem demonſtratur, ſi circulus LZG (Fig. 8. n. 2.) ſit minor circulo KCT, nam, ducta adhuc AI, triangulum LAH eſt æquale, & æquiangulum triangulo PAI; adeoque eſt arcus XH = GI, ſimilis QC, ſimilis QA; ergo, ſi ducatur QE, angulus AEQ = GPI; ſed anguli AEQ, & AGQ, item anguli GPI, & GPK ſunt æquales duobus rectis; ergo angulus GPK = AGQ, & linea GQ eſt parallela PK. Quia vero in triangulis AGQ, & APK angulus AGQ = APK, QAG = PAK, & latus AG = AP; ergo AQ = AK, & linea PI producta tranſit per punctum K.

Corollarium I.

Ducta ATM (Fig. 8. n. 1.) a centro A per punctum T, quia triangulum AKT iſoſceles, erit AKP = ATI; ſed etiam APK = AIT, & latus AP = AI; ergo triangula AKP, ATI, & QAG ſunt æqualia, & æquilatera. Cum igitur ſit linea TI æqualis, & parallela QG, ſi intelligantur ductæ chordæ arcuum QT, & GI, erunt hæ æquales, & parallelæ; cum autem chorda arcus GI ſit æqualis radio circuli LZG, erit etiam chorda arcus QT æqualis eidem radio.

Corollarium II.

Cum in triangulis LAB, IAK angulus ALB ſit æqualis AIK, angulus LAB communis, latus AL = AI,

erit triangulum LAB = IAK, & latus AB = AK; cum igitur AK ſit radius, etiam AB erit radius, hoc eſt duæ lineæ LH, AI ſe mutuo ſecant in puncto B communi circumferentiæ circuli KTC.

Corollarium III.

Quia triangulum IAK = PAT, erit etiam PAT = LAB; ſed totum triangulum PAI = LAH; ergo reſiduum ATI = ABH; eſt autem ATI æquiangulum ABH; ergo linea LH ſecat bifariam arcum TC.

Corollarium IV.

Cum TB ſit æqualis BC, erit etiam MI = IH. Cum igitur LH ſit duplus IG, & MH duplus IH, erit LM duplus GH; ſed quia arcubus LM, HG ſunt ſimiles arcus KT, FC, etiam KT erit duplus FC, ac proinde ZG duplus KT, hoc eſt arcus ZEG circuli AZE ſectus a circulo LMG ſemper duplus erit arcus KT circuli KTC ſecti a linea PI.

Corollarium V.

Si ab æqualibus lineis LH, PI demantur æquales BH, TI, LO, PK, reſiduæ KT, OB erunt æquales; quare arcus KOT = OTB = GE = EZ; ſed quia arcus

cus QE eſt duplus AS, & arcus QG duplus AV, erit GE, ac proinde KOT, vel OTB duplus VS.

Corollarium VI.

Quoniam angulus pAR = RAE = QAG, eſt arcus pR = RE = QG = SZ; cum ergo ſit SZ = Rp, detracto pZ, erit Sp = ZR; & cum SZ = ER, utrique addito ZR, erit SR = ZE = GE = KT = OB; ſed arcus SR eſt duplus SB; ergo etiam arcus OB eſt duplus SB; ergo arcus OS = SB = SV. Cum igitur arcus QES ſit duplus QS per conſtructionem, arcus SR ſit duplus SV ex dictis, etiam arcus QR erit duplus QV.

Corollarium VII.

Quia menſura anguli NmD eſt $\frac{1}{2}$ DN — $\frac{1}{2}$ yo; & $\frac{1}{2}$ TB + $\frac{1}{2}$ KO; erit arcus DN — yo = ER duplus TB; & quia menſura anguli AHy eſt $\frac{1}{2}$ Ay + $\frac{1}{2}$ EN, eſt $\frac{1}{2}$ AS; ergo Ay + EN = AS; &, dempto Ay, eſt yS = EN, & Qy = QN.

Corollarium VIII.

Cum AV ſit æqualis arcui KO, ſi a puncto Q per V ducitur QV, hæc concurrit in O, ubi linea LH ſecat

cat circulum KBC; & triangula LQV, LAG, QAO, ducto radio AO, ſunt iſoſcelia, & ſimilia.

Corollarium IX.

Si ducatur linea OS, item SV, quæ protrahatur in r, addita rQ, duo triangula OVS, rVQ ſunt æquilatera; nam triangulum VOS eſt iſoſceles, quia SO = SV; ergo angulus SOV = SVO; ſed SOV eſt 60°; ergo etiam SVO eſt 60°. Patet etiam triangulum rVQ eſſe æquilaterum; nam angulus rQO = rSO in eodem circuli ſegmento, & anguli ad V ſunt ad verticem oppoſiti.

Corollarium X.

Si producitur linea QB in h, & additur Sh, item SB producta in n, item QN, etiam triangula SBh, QBn ſunt æquilatera; nam, cum menſura anguli OQh ſit arcus OB, ſit arcus Vh, erit OB = Vh; ſed OS = VS; ergo arcus SB = Sh, & chorda unius æqualis chordæ alterius; angulus autem BhS eſt 60°; eſt ergo OV = OS = SV = SB = Sh = hB, & rV = VQ = Qr = QB = Qn = Bn.

Corollarium XI.

Si a puncto O ad h ducitur Oh, hæc eſt parallela VB; eſt enim QO : Qh = QV : QB.

Corol-

Corollarium XII.

Quia arcus SR eſt duplus arcus SB; arcus autem SB = Sh, erit Sh = hR; linea ergo Qh ſecat bifariam arcum SR; ac proinde, ſi a puncto h per centrum C ducitur ih, iſta ſecabit bifariam, & ad angulos rectos chordam ductam a puncto S ad R; item huic parallelas oD, & QG. Linea etiam Sn ſecat bifariam arcum GE; nam, quia angulus QBn eſt 60°, eſt arcus Qn + Sh = Qn + nE; ergo, dempto Qn, Sh = nE; ſed GE eſt duplus Sh; ergo GE eſt duplus nE; adeoque Gn = nE.

Corollarium XIII.

Producta linea OS in s, angulus VSs eſt duplus anguli rSO; ergo arcus VQs eſt duplus rO, eſt duplus QS; ſed QE = QS; ergo QV + Es = QS, &, dempto QV, Es = VS. Linea ergo Ss ſecat bifariam arcum ZE; eſt ergo Gn = nE = Es = Zs = hR = Sh = SV; ac proinde Rs = hZ = ph.

Corollarium XIV.

Quia eſt arcus QS = GR, erit SR + QG = QS; eſt igitur arcus QG circuli AZE interceptus inter utramque communem ſectionem Q, & G complementum

120°

120° arcus SR eiuſdem circuli intercepti inter communem ſectionem S, & lineam AR ductam a centro A per punctum I, ubi linea PD ſecat peripheriam circuli LMH.

Corollarium XV.

Ex Corollario ſeptimo triangulum QNy eſt iſoſceles; ergo rectangulum $NL \times Ly + Qy^2 = LQ^2$; ſed $LQ^2 = QL \times LA + LQ \times QA$; & $QL \times LA = NL \times Ly$; ergo $Qy^2 = LQ \times QA$; ſed rurſus $Qy^2 = NB \times By + QB^2$; quia triangulum NQy eſt iſoſceles, & $NB \times By + QB^2 = QB \times Bh + QB^2 = hQ \times QB$; ergo $hQ \times QB = Qy^2$; ergo $hQ \times QB = LQ \times QA$. *Corollaria hactenus tradita, ſi pauca in aliquibus demonſtrationibus immutentur, vera ſunt etiam in Figura* 8. *n.* 2.

Corollarium XVI.

Sint modo (Fig. 9.) circuli KTC, & AZE æquales, ut in Fig. 8., & fiat $QG = AQ$. Ducatur AG a puncto A ad G, & centro A radio AG deſcribatur circulus LMG, fiantque omnia, ut in Figura 8. Cum per conſtructionem ſit $QG = AQ$, erit æqualis $GE = EZ$, hoc eſt circumferentia circuli AZE erit diviſa in ſex partes æquales. Igitur, cum GE debeat eſſe æqualis KT, linea PI, quæ tranſit per punctum K, tranſibit

etiam

etiam per communem ſectionem S; adeoque quatuor diverſa puncta S, p, T, o, Figuræ 8. concurrunt in unum punctum S. Similiter, cum arcus SR debeat eſſe æqualis KT, & hic ſit æqualis SZ; ergo SR = SZ, hoc eſt quatuor puncta I, Z, R, D, quæ in Figura 8. ſunt diſtincta, in unum punctum Z concurrent.

Corollarium XVII.

Cum arcus QEZ ſit ſemicirculus, & linea AI concurrat in Z, angulus QAZ eſt rectus, & arcus QB eſt 90°. Eſt ergo triſectus angulus, & arcus 90°.

Corollarium XVIII.

Quia arcus AS = GE, & yS = EN, erit Ay = GN; adeoque erit NL parallela PG.

Corollarium XIX.

Ducatur a centro C ad punctum V, ubi linea GL ſecat arcum AZ, linea CV ſecans AB in q. Cum arcus AV ſit æqualis CB, erit triangulum AqC iſoſceles; cumque angulus ACV ſit æqualis AHL, erit Cq parallela HL; Triangula ergo LAH, ABH, AqC ſunt ſimilia, & iſoſcelia; eſt igitur LH : AH = AH : AB = AB, ſeu AC : Aq. Sunt ergo lineæ AB, & AH duæ mediæ proportionales inter duas lineas Aq, & LH.

S Corol-

Corollarium XX.

Ex dictis patet circulum KBC ſecare in tres partes æquales utramque lineam LH, PI in O, & B, in K, & T. Patet etiam circulum LMG triplum eſſe circuli KBC; nam, ob triangulum æquilaterum AOB, rectangulum OHB + $AB^2 = AH^2$; ſed rectangulum OHB = $2AB^2$, quia OB = BH = AB; ergo $AH^2 = 3AB^2$; ſed circuli ſunt, ut quadrata radiorum; ergo circulus LMG eſt triplus KBC.

Corollarium XXI.

Ducta chorda AV, triangula GLA, QVL, QAO, AVO, AVS, OSB, OSh, quæ quidem quatuor ultima ſunt æqualia, & OmS ſunt iſoſcelia, & ſimilia; eſt ergo QO : AO = AO : OV = OV, ſeu OS : Om. Patet ergo AO eſſe tangentem circuli AZE in A, Oh eſſe tangentem utriuſque circuli KBC, AZE, & hC eſſe tangentem circuli KBC in C, inſuper AOhC eſſe quadratum radii.

Corollarium XXII.

Triangula QOh, QVB, BOh ſunt iſoſcelia, & ſimilia. Facile etiam demonſtratur lineam VB utrinque productam concurrere in s, & a, ubi lineæ Os, AP ſecant

ſecant circulos AZE, & KBC: Item lineam VC utrinque productam concurrere in n, & K, & tam lineam BV productam in a, quam lineam CV productam in K æquales eſſe radio AH.

Corollarium XXIII.

Lineæ LH, AH, AC, Aq, quas eſſe continuo proportionales demonſtravimus Corollario XIX. ſuppoſito radio AC = 1, ſunt inter ſe, ut hi numeri $3 : \sqrt{3} : 1 : \frac{1}{\sqrt{3}}$.

Corollarium XXIV.

Cum demonſtratum ſit Corollario XXI. quatuor lineas QO, AO, OS, Om eſſe continuo proportionales, erit ſubtenſa QO 150° ad radium AO, ut radius AO ad ſubtenſam OS 30°, ut ſubtenſa OS 30° ad lineam Om. Quare cum ex tabulis ſinuum, & tangentium notæ ſint lineæ QO, AO, OS, inveniri poterit arcus, cuius chorda ſit linea Om.

Corollarium XXV.

Sit Figura decima deſcripta modo, quo eam dixi eſſe deſcribendam pro ſolutione problematis de Enneagono. Cum demonſtratum ſit Corollario V. Theor. I. lineam PTI tranſire per punctum K, & eſſe paralle-

lam QG, ſitque hæc æqualis LH, ductis omnibus lineis, ut in Figura 8. & 9., puncta p, h, M, Z Figuræ octavæ concurrent in unum punctum Z; cumque ibi demonſtratus fuerit arcus Sp = ZR, arcus SZ = ER, erit in Figura decima arcus SZ = ZR = RE = En = Gn = QG = VS.

Corollarium XXVI.

Quia arcus Zo = ZD, & arcus ZS = ZR, erit reſiduus So = RD = EN; erit ergo arcus ER = DN, & arcus RN = ZD.

Corollarium XXVII.

Cum arcus RN ſit æqualis ZD, duæ lineæ æquales LH, & PI ſe mutuo ſecabunt in puncto communi peripheriæ circuli AZE; nam angulus ABO eſt æqualis ATK; menſura autem anguli ABO = $\frac{1}{2}$ RN + $\frac{1}{2}$ Am, & anguli ATK = $\frac{1}{2}$ ZD + $\frac{1}{2}$ Ao; ſed $\frac{1}{2}$ RN = $\frac{1}{2}$ ZD; ergo $\frac{1}{2}$ Am = $\frac{1}{2}$ Ao; ergo integer arcus Am = Ao; ergo puncta m, & o coincidunt in idem punctum circumferentiæ AZE.

Corollarium XXVIII.

Triangula LVO, OSB ſunt iſoſcelia, & æqualia, & ſunt ſimilia triangulis Lωo, & muB. Item triangula

LVr,

LVr, BOq ſunt æqualia, & iſoſcelia, & ſunt ſimilia triangulis LBZ, mBt, rBS.

Corollarium XXIX.

Linea OZ bifariam ſecat utrumque arcum SO, & SZ; patet ergo eſſe Ox : OS = OS : OZ, hoc eſt chordam 20° eſſe ad chordam 40°, ut ſubtenſa 40° ad lineam OZ.

Corollarium XXX.

Quia LORZ eſt parallelogrammum, linea LZ = OR = QZ; ſumma ergo chordarum 80°, & 40° circuli AZE æqualis eſt chordæ 160° eiuſdem circuli. Quod ſi conſulamus Figuram 9. conſtabit chordam Qh 150° eſſe æqualem ſummæ chordarum QB 90°, & Bh 30°; & ſi advertamus Corollario XXVI. Theor. I. conſtabit in Fig. 6. chordam me, vel Qy 140° æqualem eſſe ſummæ chordarum QT 100°, & Ty 20°. Generatim quælibet linea Qh in Fig. 8., erit ſumma chordarum QB arcus QCB, & BS arcus BTS, qui arcus ſimul ſemper efficiunt 120°. Hinc ſuppoſito arcu CS diviſo in omnes ſuos gradus facile erit deducere

Summam chordarum arcuum

120.° + 0° = 120°
119. + 1. = 121
118. + 2. = 122
117. + 3. = 123
116. + 4. = 124 &c.

Corol-

Corollarium XXXI.

Quia triangula BnR, RAn ſunt iſoſcelia, erit An^2, ſeu $AR^2 = RA \times AB + Bn^2$; ſed $AR^2 = RA \times AB + AR \times RB$; ergo $RA \times AB + Bn^2 = RA \times AB + AR \times BR$; ergo $Bn^2 = AR \times BR$. Eſt igitur chorda AR 140° ad Bn 80°, ut Bn 80° ad BR.

Scholion I.

In Corollario X. demonſtravi triangulum SBh (Fig. 8. 9. 10.) eſſe æquilaterum, adeoque eſſe $Sh = Bh$; ſed in Corollario XII. demonſtravi arcum $Sh = hR$; ergo erit chorda $Sh = Bh = hR$; ergo erit triangulum BhR iſoſceles. Cum autem in hypotheſi Figuræ exhibentis ſolutionem Problematis de Enneagono per Corollarium XXV. puncta h, & Z in idem concurrant, triangulum BhR in Figura 10. non erit diverſum a triangulo BZR, & chorda Ah eadem erit, ac chorda AZ. Quia igitur triangulum BZR eſt iſoſceles, erit $RA \times AB + BZ^2 = AZ^2$; ſed $RA \times AB = AB^2 + AB \times BR$; ergo $AB^2 + AB \times BR + BZ^2 = AZ^2$. Dicatur autem $AB = a$, ſi chorda AZ ea eſſet, quam ponit Auctor ſolutionis de Enneagono circulo inſcribendo, eſſet æqualis $a\sqrt{\frac{7}{3}}$, adeoque $AZ^2 = \frac{7a^2}{3} = AB^2 + AB \times BR + BZ^2$; ſed

AB²

$AB^2 = a^2$; ergo $\frac{7a^2}{3} = a^2 + AB \times BR + BZ^2$, & $\frac{4a^2}{3} = AB \times BR + BZ^2$; ſed $AB \times BR + BZ^2 = QZ \times BZ$; ergo $\frac{4a^2}{3} = QZ \times BZ$; ergo $\frac{4a^2}{3QZ} = BZ$. Eſt autem per Corollarium XV. $LQ \times QA = Qh \times QB$, hoc eſt $= QZ \times QB$, quia puncta h, & Z concurrunt in unum; ergo eſt $QB : LQ = QA : QZ$; ſed $QB = ZE = a\sqrt{\frac{5}{3}}$ per ſupradictum Auctorem, $LQ = a\sqrt{\frac{7}{3}} + a$; $QA = a$; ergo $a\sqrt{\frac{5}{3}} : a\sqrt{\frac{7}{3}} + a = a : QZ$; ergo $QZ = \frac{a\sqrt{\frac{7}{3}} + a}{\sqrt{\frac{5}{3}}}$; ergo $QZ - QB = BZ = \frac{a\sqrt{\frac{7}{3}} + a}{\sqrt{\frac{5}{3}}} - a\sqrt{\frac{5}{3}} = \frac{4a\sqrt{\frac{5}{3}}}{3\sqrt{\frac{7}{3}} + 3}$, ſuppoſito, quod puncta h, & Z concurrant in unum; ſed $\frac{a\sqrt{\frac{7}{3}} + a}{\sqrt{\frac{5}{3}}} - a\sqrt{\frac{5}{3}}$ eſt quantitas minor quantitate $\frac{4a\sqrt{\frac{5}{3}}}{3\sqrt{\frac{7}{3}} + 3}$; ergo puncta h, & Z non concurrunt in unum; ergo linea aſſumpta ab Auctore ſolutionis Problematis de Enneagono non eſt ea, quæ ſolvit Problema. Patet igitur independenter a tabulis trigonometricis, ut ab initio promiſeram, probari addu-

adductam ſolutionem de Enneagono circulo inſcribendo non eſſe exactam.

Scholion II.

Si radius AC (Fig. 9.) dicatur æqualis a, erit $LQ = a\sqrt{3} + a$, & $LQ \times QA = a^2\sqrt{3} + a^2 = Qy^2 = hQ \times QB$; erit igitur $Qy = \sqrt{a^2\sqrt{3} + a^2}$, $\frac{Qy}{2} = \frac{\sqrt{a^2\sqrt{3} + a^2}}{2}$. Similiter, quia $QB = a\sqrt{2}$, erit $Qh = \frac{a\sqrt{3} + a}{\sqrt{2}}$, & $\frac{Qh}{2} = \frac{a\sqrt{3} + a}{2\sqrt{2}}$, & $Bh = Qh - QB = \frac{a\sqrt{3} + a}{\sqrt{2}} - a\sqrt{2} = \frac{a\sqrt{3} - a}{\sqrt{2}}$, & $\frac{Bh}{2} = \frac{a\sqrt{3} - a}{2\sqrt{2}}$. Quare, ſi ponatur $a = 10000000$, $\sqrt{3} = 1.7320508$, $\sqrt{2} = 1.4142135$, habebitur $Qy = \sqrt{a^2\sqrt{3} + a^2} = 16528916$, & $\frac{Qy}{2} = 8264458$ ſinus dimidii arcus QAy, qui ſinus in tabulis reſpondet proxime ſinui arcus 55° 44′ 7″; pariter erit $Qh = \frac{a\sqrt{3} + a}{\sqrt{2}} = 19318517$, & $\frac{Qh}{2} = 9659258$, qui eſt ſinus arcus 75°; tandem $Bh = \frac{a\sqrt{3} - a}{\sqrt{2}} = 5176381$, & $\frac{Bh}{2} = 2588190$, qui eſt ſinus arcus 15°.

Scholion III.

Quia $Qh = \frac{a\surd 3 + a}{\surd 2}$, & $Bh = \frac{a\surd 3 - a}{\surd 2}$, erit $Qh + Bh = \frac{a\surd 3 + a}{\surd 2} + \frac{a\surd 3 - a}{\surd 2} = \frac{2a\surd 3}{\surd 2}$, & $\overline{Qh + Bh}^2 = \frac{12a^2}{2} = 6a^2$; eſt igitur quadratum ſummæ chordarum 150°, & 30° duplum quadrati radii AL, ſeu AG, qui latus eſt trianguli æquilateri circulo AZE inſcripti. Quod ſi intelligatur ducta linea LZ, quæ eſt latus quadrati circulo LZG inſcripti, hæc eadem linea exhibebit ſummam chordarum $Qh + Bh$; adeoque eius quadratum $LZ^2 = 6a^2$; ſed $LZ^2 = NL \times Ly + Zy^2$; quia, ſi ſupponantur ductæ chordæ ZN, & Zy, triangulum NZy eſt iſoſceles, & $NL \times Ly = QL \times LA$; ergo $LZ^2 = QL \times LA + Zy^2$, ſeu $6a^2 = 3a^2 + a^2\surd 3 + Zy^2$; adeoque $Zy^2 = 3a^2 - a^2\surd 3$, & $Zy = a\sqrt{3 - \surd 3} = 11260325$, $\frac{Zy}{2} = 5630162$, qui ſinus eſt proxime arcus 34° 15′ 53″.

THEOREMA III.

Sint duo circuli KNC, & ANE (Fig. 11.) æquales ita deſcripti, ut peripheria unius tranſeat per centrum

trum

trum alterius; ſi a communi ſectione Q per centrum A ducetur QAK, & a puncto K per quodlibet punctum T arcus AN linea KTG; inſuper ſi a centro A per ſectionem T ducetur AR fietque arcus KR = RO, & agetur BO uſque ad Z, additis AB, AOM, AZ, ASD, & AG, triangula KAS, & BAO erunt inter ſe æquiangula, & æqualia, item KAG, & BAZ, item ASG, & AOZ.

Demonſtratio.

Quia arcus KR per hypotheſim eſt æqualis RO, & AR radius, eſt KT = TO. Cum igitur in triangulis KAT, OAT ſit KT = OT, AT commune, AK = AO, hæc ſunt æquiangula; eſt ergo angulus AOB = AKS = ABO = ASK; ergo etiam triangula BAO, KAS ſunt æqualia, & æquiangula. Q. e. p.um. Sed etiam in triangulis KAG, & BAZ angulus AGK = AZB, AKG = ABZ, latus KA = AB; ergo duo triangula AKG, & ABZ ſunt æquiangula, & æqualia, Q. e. ſec.dum. Tandem, quia in triangulis ASG, & AOZ angulus AGS = AZO, GAS = ZAO, cum totus angulus GAK ſit æqualis toti ZAB, & ablatus SAK = OAB ablato, & latus AS = AO; ergo triangula ASG, & AOZ ſunt æquiangula, & æqualia. Q. e. t.um.

Corol-

Corollarium I.

Quia chorda AZ eſt æqualis AG, arcus ANZ eſt æqualis AQG, adeoque etiam reſiduus ZE æqualis eſt reſiduo GE, quamobrem diameter AE ſemper ſecat bifariam arcum ZG inter utramque lineam KG, & BZ interceptum.

Corollarium II.

Patet arcum XO æqualem eſſe yS, &, utrique addito XS, arcum Xy æqualem eſſe SO. Pariter MZ æqualem eſſe GD, MD = GZ; item ZE = SO = Xy.

Corollarium III.

Cum ZE ſit dimidium ſummæ arcuum ZD + GD, & ZE + ED ſit arcus maior, GE — ED arcus minor; ergo ED erit ſemidifferentia inter utrumque arcum. Portio igitur arcus intercepta inter diametrum AE, & lineam AS productam in D erit ſemidifferentia inter arcum ZD, & arcum GD.

Corollarium IV.

Si a puncto Q per punctum T ducatur QP, erit arcus KP = AT, & PN = TN; ſed TN duplus RN;

ergo PR = RN; ergo KP = NO, & TN = OC; eſt igitur portio arcus OC inter BZ, & AE æqualis arcui TN, & PR eſt ſemidifferentia inter arcum OP, & KP. Arcus autem OP, OR, ON ſunt in proportione arithmetica; nam OP + ON, ſeu OP + PK = 2OR.

Corollarium V.

Conſtat eſſe in proportione arithmetica arcum QE : QO = QO : EM; item NZ : AN = AN : ZE; ſimiliter ZE : OC = OC : MZ; pariter GD : yC = yC : DE; eſt enim QE + EM = 2QO, NZ + ZE = 2AN, ZE + MZ = 2OC, tandem GD + DE = 2yC.

Corollarium VI.

Cum per Corollarium III. ED ſit ſemidifferentia inter arcum GD, & ZD, & eadem ED ſit differentia inter ZE, ſeu GE, & GD, patet, quod evaneſcente hac differentia, hoc eſt congruente linea AD diametro AE, arcus GD erit æqualis GE = ZE = MZ; adeoque arcus MG erit triſectus.

Corollarium VII.

Sint ergo duo circuli æquales KNC, & ANE (Fig. 12.), & ducta linea QAK, a puncto K per centrum C ducatur KCG, & reliqua fiant, ut in ſuperiori

Fi-

Figura. Tunc AD congruet AE; adeoque nulla erit differentia DE; erit ergo arcus MG triſectus. Eſt autem arcus MG 90°; eſt igitur triſectus arcus 90°. Pariter Oy eſt 45°, & eſt triſectus; QET 270° eſt triſectus, & Ky 135° eſt triſectus.

Corollarium VIII.

Quia QT eſt latus quadrati circulo ANE inſcripti, erit eius quadratum æquale dimidio quadrato diametri, & erit æquale rectangulo QKA; ſed rectangulum QKA eſt æquale GKT; ergo quadratum QT eſt æquale rectangulo QKA, rectangulo GKT, & QK : QT = QT : QA; GK : QT = QT : KT.

Corollarium IX.

Triangula GKA, QTK, ATQ, AFC ſunt ſimilia; cum igitur ſit KQ : QT = QT : QA = QA, ſeu AC : AF, quatuor lineæ KQ, QT, QA, AF erunt continuo proportionales. Quod ſi diameter QK ponatur æqualis 2, ſupradictæ lineæ erunt, ut hi numeri $2 : \sqrt{2} : 1 : \frac{1}{\sqrt{2}}$. Patet rectam AF eſſe ſinum rectum arcus 45°, eſt enim dimidium chordæ QT 90°.

Corol-

Corollarium X.

Cum triangulum KAC ſit iſoſceles, erit rectangulum $KFC + AF^2 = AC^2$. Suppoſito ergo radio $AC = 1$, cum ſit per Corollarium præcedens $AF^2 = \frac{1}{2}$, etiam rectangulum $KFC = \frac{1}{2}$. Eſt igitur $KF : AF = AF : FC$.

Corollarium XI.

Triangula KAC, AnC, TnO ſunt ſimilia; inſuper triangulum $AnC = BAu$, & $TnO = KuT$, & ſunt iſoſcelia. Eſt ergo $nC = nA = uA = uB$, & $nO = nT = uT = uK$. Eſt vero $KC : AC = AC : Cn$; ergo, quia $KC = \sqrt{3}$, $AC = 1$, erit $Cn = nA = uA = uB = \frac{1}{\sqrt{3}}$, & $nO = nT = uT = uK; = 1 - \frac{1}{\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}}$; $Gn = Qu = 1 + \frac{1}{\sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{3}}$; $Kn = uO = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$; $KT = TO = OM = \sqrt{3} - 1$.

Corollarium XII.

Quia $KT : KA = KQ : KG$, hoc eſt $\sqrt{3} - 1 : 1 = 2 : \frac{2}{\sqrt{3}-1}$, erit $KG = \frac{2}{\sqrt{3}-1}$, & $\frac{KG}{2} = KF = \frac{1}{\sqrt{3}-1}$; ſed

ſed $KF \times FC = AF^2 = \frac{1}{2}$ per Corollarium X. ergo $FC = \frac{\sqrt{3} - 1}{2}$. Eſt igitur per Corollarium præcedens $Kn = \frac{2}{\sqrt{3}}$ dupla $Cn = \frac{1}{\sqrt{3}}$, & $KT = \sqrt{3} - 1$ dupla $FC = \frac{\sqrt{3} - 1}{2}$; ergo etiam $Tn = \frac{\sqrt{3} - 1}{\sqrt{3}}$ erit dupla nF, quæ propterea erit æqualis $\frac{\sqrt{3} - 1}{2\sqrt{3}}$.

Corollarium XIII.

Ex dictis patet rectangulum KnC æquale eſſe GnT; quadratum An æquale eſſe GnF; quadratum AF æquale GFC.

Corollarium XIV.

Linea KTG ſecans arcum AN modo ſecare poteſt diametrum AE inter A, & C, modo inter C, & E, & modo poteſt eſſe tota ſupra diametrum AE, ut patet in Figuris 11. 13. 14. Pro diverſitate autem poſitionis lineæ KTG etiam linea BTZ modo eſt tota ſupra diametrum AE, & modo eandem ſecat. Eſt igitur primo ſemper verum, quod diameter AE ſecat bifariam arcum ZG inter utramque lineam BTZ, KTG ſitum, quemadmodum dixi Corollario I.; ſecundo verum eſt arcum XO æqualem eſſe Sy in Figura 13., & detra-

cto

cto Oy, arcum SO = Xy; ac proinde etiam DG = ZM, &, ablato MG, arcum MD = ZG; pariter verum eſt etiam in Figura 14. arcum XO = Sy, &, dempto SX, arcum SO = Xy; atque ita etiam MZ = GD, & MD = GZ; tertio, quia in Figura 13. ZD eſt arcus maior, & GD minor, portio arcus inter utramque lineam KTG, & BTZ erit differentia utriuſque arcus; cumque ſit ZG biſectus a diametro AE, erit GE, vel ZE eorundem arcuum ſemidifferentia; ſimiliter, quia GD eſt arcus maior in Figura 14. ZD minor, GZ eſt differentia, & ZE ſemidifferentia; quarto in omnibus Figuris vera ſunt, quæ adnotavi Corollario IV.; quinto, quia in unaquaque Figura arcus GE, ſive ZE ſemper eſt æqualis OS, erit in Figura 13. differentia inter ZE, & ZM æqualis EM, quæ eſt dupla OC; ergo tres arcus ZM, SC, & ZE ſunt in proportione arithmetica continua; item in Figura 14. differentia inter ZE, & OC æqualis SC, inter GD, & ZE æqualis ED, dupla SC; ergo tres arcus GD, OC, & ZE ſunt in proportione arithmetica continua.

Corollarium XV.

Si linea KTG (Figura 15.) ducitur ad extremum E diametri AE, tunc evaneſcit ſemidifferentia GE, adeoque etiam ſemidifferentia ZE; ergo duæ lineæ KTG,
BTZ

BTZ congruunt, atque unam lineam efficiunt. Ducta igitur linea ATR, hæc biſecabit arcum KO, & erit perpendicularis lineæ KTO; quare erit linea KT ſinus rectus arcus KR.

Corollarium XVI.

Ducta a puncto Q per O linea QOH, quia triangulum KQE eſt rectangulum, & KQO pariter rectangulum, erunt tria triangula KQE, KQO, QOE ſimilia; ergo erit KE : KQ = KQ : KO, item KE : QE = QE : EO, & KO : QO = QO : OE. Suppoſita autem diametro = 2, erit $KE = \sqrt{7}$, & KQ = 2; ergo $KO = \frac{4}{\sqrt{7}}$, $OE = \frac{3}{\sqrt{7}}$, $KT = TO = \frac{2}{\sqrt{7}}$, $ET = \frac{5}{\sqrt{7}}$.

Corollarium XVII.

Cum triangula EAK, QTK ſint ſimilia, erit KA : AE = KT : TQ, hoc eſt $1 : 2 = \frac{2}{\sqrt{7}} : \frac{4}{\sqrt{7}}$; ergo chorda QT = KO. Eſt autem rectangulum $KT \times TO = KT^2 = QT \times TP$; ergo QT : KT = KT : TP, hoc eſt $\frac{4}{\sqrt{7}} : \frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} : \frac{1}{\sqrt{7}}$; ſed PT æqualis eſt chordæ arcus PN, adeoque, $\frac{PT}{2} = \frac{1}{2\sqrt{7}}$ æqualis eſt ſinui recto arcus PR; ergo $QT : KT = KT : TP = TP : \frac{TP}{2}$. Sunt

igitur in proportione geometrica continua quatuor ſupradictæ lineæ, ut ſunt in proportione arithmetica continua $ET : QT = QT : EO = EO : OT = OT : TP$; nimirum $\frac{5}{\sqrt{7}} : \frac{4}{\sqrt{7}} = \frac{4}{\sqrt{7}} : \frac{3}{\sqrt{7}} = \frac{3}{\sqrt{7}} : \frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} : \frac{1}{\sqrt{7}}$. Patet etiam chordam QP æqualem eſſe TE.

Corollarium XVIII.

Quia triangulum QKO eſt ſimile AKT, chorda QO erit dupla AT; ſed $AT^2 = AK^2 - KT^2 = 1 - \frac{4}{7} = \frac{3}{7}$; ergo $AT = \frac{\sqrt{3}}{\sqrt{7}}$; ergo $QO = \frac{2\sqrt{3}}{\sqrt{7}}$. Sed, ob rectangulum $QO \times OH = EO \times OT$, eſt $QO : EO = OT : OH$, ſeu $\frac{2\sqrt{3}}{\sqrt{7}} : \frac{3}{\sqrt{7}} = \frac{2}{\sqrt{7}} : \frac{3\sqrt{7}}{7\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{7}}$; ergo $QH = \frac{3\sqrt{3}}{\sqrt{7}}$; & QO eſt media arithmetica inter QH, & OH.

Corollarium XIX.

Cum rectangulum $EO \times OT$ ſit æquale rectangulo $AO \times OD$, & ſit $EO \times OT = \frac{2}{\sqrt{7}} \times \frac{3}{\sqrt{7}} = \frac{6}{7}$; erit pariter $AO \times OD = \frac{6}{7}$; ſed $AO = 1$; ergo $OD = \frac{6}{7}$.

Corol-

Corollarium XX.

In hac Figura, & in omnibus ſuperius explicatis arcus Q A T ſemper eſt æqualis arcui K R O, & arcui T N M; nam, quia totus arcus QN eſt duplus KN, & ablatus T N eſt duplus R N, etiam reſiduus QAT eſt duplus KR; ſed KR = RO; ergo QAT = KRO; ſed etiam TNM eſt duplus RO; ergo QAT = KRO = TNM.

Corollarium XXI.

In omnibus Figuris verum eſt arcus QS, Qy, & A T eſſe in proportione arithmetica; nam in triangulo KAG menſura anguli AGK eſt arcus $\frac{AT}{2}$, & anguli AKG eſt arcus $\frac{QS}{2}$; ſed anguli AGK, & AKG æquales ſunt externo QAy; ergo utriuſque anguli menſura eſt arcus Qy; Eſt igitur QS + AT = 2Qy; ergo eſt QS : Qy = Qy : AT.

Corollarium XXII.

Cum per Corollarium XX. arcus QAT ſemper ſit æqualis arcui TNM, etiam chorda unius arcus ſemper erit æqualis chordæ alterius. Si ergo, data linea QAK

per centrum A in duobus circulis KNC, & ANE, a puncto K ducatur KTG, ita ut TQ ſit æqualis TG, tunc conſtructa Figura 16. eodem modo, quo reliquæ ſuperiores, lineæ AOM, & AyG unam lineam efficient, hoc eſt puncta M, & G erunt idem punctum; ſed per Corollarium XIV. arcus MD eſt æqualis ZG; ergo arcus DG erit duplus GE; ergo arcus DE, item arcus SC erit triſectus.

Corollarium XXIII.

Patet in Figuris 13. & 14., quod ſi arcus MG evadat æqualis GE, vel arcus DZ = ZE, tunc arcus DG, ſeu MZ, in Figura 13., item arcus MZ, ſive DG in Figura 14. erit triſectus.

Corollarium XXIV.

Si fuerit arcus ZE duplus TN in Figura 16., vel arcus TN duplus GE in Figura 17., arcus DE, & SC in primo caſu, & arcus MZ, vel GD, ſeu OX, & Sy in ſecundo caſu erit triſectus; etenim ex dictis patet arcum ZE eſſe æqualem SO, & arcum TN = OC; ergo, ſi ZE eſt duplus TN, etiam SO erit duplus OC; adeoque arcus SC, ac proinde etiam DE erit triſectus, & linea AO producta concurret in G. Item in ſecundo caſu, ſi TN eſt duplus GE, eſt etiam duplus OS;

ſed

ſed OC = TN; ergo OC eſt duplus OS; ergo OS = SC = Xy; &, dempto XC, SX = Cy; ſed Cy = CX; ergo arcus SX = CX = Cy; ac proinde etiam DZ = ZE = GE.

Scholion I.

Facile deducitur (in Figuris 11. 12. 13. 14. 15. 16. 17.) rectangulum GKT conſtanter eſſe æquale rectangulo QKA, ſeu quadrato lateris quadrati circulo ANE, vel KNC inſcripti; inſuper facile infertur rectangulum GKT eſſe ad rectangulum ZBT, ut linea KT ad lineam BT; etenim lineæ KG, & BZ ſemper ſunt inter ſe æquales. Hinc data linea KT ſemper nota erit KG, item TG; ponatur ſiquidem diameter QK = 2a, & KT = b, erit $\frac{2a^2}{b}$ = KG, & $\frac{2a^2}{b} - b$ = TG; quare nota erit chorda arcus TG.

Scholion II.

Conſtat in Figura 12. lineam KT eſſe omnium minimam, quæ duci poſſit a puncto K ad peripheriam convexam ANE; eſt enim ea, quæ eſt in directum cum linea, quæ tranſit per centrum C circuli ANE; Erit pariter linea KG omnium maxima, quæ duci poſſit a puncto K ad circumferentiam concavam circuli ANE, quia tranſit per centrum. Cum itaque per Corol-

rollarium XI. linea KT ſit æqualis $\sqrt{3} - 1$, linea KG omnium maxima erit æqualis $\sqrt{3} + 1$.

Scholion III.

In Figura 16., quia arcus QE eſt duplus AN, & ZE ſupponitur duplus TN in Corollario XXIV.; eſt QZ duplus AT; productis ergo lineis QK, ZT, donec concurrant in V, triangulum AVZ erit iſoſceles, & circulus centro A radio AV deſcriptus tranſibit per puncta G, & Z. Licebit igitur centro A, radio AV circulum deſcribere, atque Figuram conſtruere eo modo, quo diximus in Theoremate ſecundo, pluraque occurrent ſcitu non iniucunda, quæ tamen non vacat expendere, quia ex dictis nullo negotio inferuntur.

THEOREMA IV.

Sint duo circuli æquales KRC, ANE (Fig. 18. n. 1. 2. 3.) ita deſcripti, ut circumferentia unius tranſeat per centrum alterius, & a communi ſectione Q, ducta diametro QAK, ducatur quælibet KG ſecans circulum KRC in aliquo puncto R; ſi a centro A ad R ducetur radius AR, & a puncto K in radium AR perpendicularis KO ſecans circulum KRC in O, atque a puncto Q per O linea QH occurrens lineæ KG in H, primo erit KH bifariam ſecta in R, ſecundo linea QH erit æqualis diametro.

De-

Demonſtratio.

Quia KO per conſtructionem eſt perpendicularis radio AR, & eadem KO eſt perpendicularis QO, quia angulus QOK eſt in ſemicirculo, erit QH parallela AR; ergo erit QK : KA = HK : KR, ac proinde erit KH bifariam ſecta in R. Q. e. p.mum; ex quo patet etiam ſec.dum; Eſt enim QH dupla radii AR. Q. e. d.

Corollarium I.

Si linea KH (Fig. 18. n. 1.) ſecabit circulum ANE in G, & g, ductis QR, QG, Qg, GZ per centrum, & Zg, & QZ, triangula RQg, QGZ erunt ſimilia; nam, quia anguli QRK, ZgG ſunt recti, utpote in ſemicirculo, erit QR parallela Zg, adeoque angulus RQg = QGZ; ſed anguli QRg, & GQZ recti; ergo triangula ſunt ſimilia, & QR : Qg = QG : GZ;

Corollarium II.

Si linea KH erit tangens circuli ANE (Fig. 18. n. 2.) ductis QR, QG, GZ per centrum, & QZ, adhuc triangula QRG, & QGZ erunt ſimilia; etenim anguli QRG, GQZ in ſemicirculo ſunt recti, & angulus QGR eſt æqualis QZG in ſegmento alterno; ergo eſt QR : QG = QG : GZ.

Corol-

Corollarium III.

Si linea KH (Fig. 18. n. 3.) dupla KR tota erit extra circulum ANE, ductis QR, QH, & a puncto g, ubi QH ſecat circulum ANE, ducta gZ parallela QR, & inſuper ZG per centrum, QG, & QZ, erit triangulum QRH = QGZ; nam erit angulus RQH = QgZ = QGZ, angulus QRH rectus æqualis GQZ recto, & diameter QH = GZ; erit igitur etiam QG = QR, & addita RG, triangulum RQG erit iſoſceles. Quia vero arcus QCR = QAG, & arcus mR = pG, erit etiam Qm = Qp; ſed arcus QCN = QAN; ergo arcus mN = pN = RN = NG, & arcus mC = Ap = QZ = KR = RO = pg, & gN = NO. Patet RG eſſe parallelam lineæ pm ductæ a puncto p ad m.

Corollarium IV.

In omnibus Figuris portio arcus Sp circuli ANE intercepta inter radium AR, ſi opus eſt, productum verſus X, & chordam QR ſemper eſt æqualis arcui pN; nam in Figura 18. n. 3. tota peripheria AZEG dupla eſt ſemiperipheriæ XCR; eſt autem arcus EZS duplus XC, & EN duplus CN; ergo, his ablatis, reſiduus SN duplus eſt reſidui RN; ſed, ob triangulum, quod ſemper conſtat eſſe æquilaterum, ductis chordis pN, & RN, arcus

arcus RN = pN; ergo pN = Sp. In Figuris vero 18. n. 1., & 2. arcus ES eſt duplus RC, & EN duplus NC; ergo reſiduus SN duplus RN, qui, cum ſit æqualis pN, hinc pN = Sp.

Corollarium V.

In unaquaque Figura linea QG ſecabit ſemper bifariam portionem arcus FN determinatam a communi circulorum KRC, & ANE ſectione N, & a puncto F, ubi diameter GZ ſecat circulum KRC extra centrum C; nam ducto radio QC in Figuris 18. n. 1., & 2. angulus GQC = QGC; ſed arcus $\frac{QC - Fm}{2}$ eſt menſura anguli QGC, & $\frac{Cm}{2}$ eſt menſura anguli GQC; ergo $\frac{QC - Fm}{2} = \frac{Cm}{2}$, QC — Fm = Cm, & QC = Cm + Fm; ſed QC = Cm + mN; ergo Cm + mN = Cm + Fm, & mN = Fm. Et in Figura 18. n. 3.; quia angulus QGC = GQC, eſt arcus $\frac{QF - Cm}{2} = \frac{Cm}{2}$, QF — Cm = Cm, & QF = 2Cm; ſed, addito utrinque arcu FC, QF + FC = 2Cm + FC = Cm + Fm = Cm + mN; ergo Cm + Fm = Cm + mN, &, dempto utrinque Cm, Fm = mN. Ducta igitur in omnibus Figuris recta mF, triangulum GmF eſt iſoſceles, ac ſimile triangulo QGC; quare eſt QG : GC = GF : Gm.

Corollarium VI.

In Figuris 18. n. 1. 2., & 3., quia linea QG ſecat bifariam arcum NF, ducta a centro A per F linea AFy, erit arcus Ny duplus NF, quadruplus Nm, quadruplus NG = Nm, ac proinde erit Gy triplus NG; ſed eſt totus circulus GAZE triplus EN; ergo, demptis Gy, GN, eſt arcus GAZy triplus GE, cumque GE ſit æqualis AZ = Qm, erit arcus GAZy etiam horum triplus.

Corollarium VII.

Cum in qualibet ſupradicta Figura demonſtratum ſit triangulum GmF iſoſceles, patet angulum externum QmF duplum eſſe interno QGZ; ergo etiam arcus QF duplus eſt QZ; ſed totus Qy duplus eſt QF, & quadruplus QZ; ergo arcus Zy eſt triplus QZ. Item arcus ZAGy eſt triplus ZE; nam totus circulus eſt triplus QE, & Zy triplus QZ; adeoque, his ablatis, reſiduus ZAGy eſt triplus ZE.

Corollarium VIII.

In quavis ex allatis Figuris cum exceſſus arcus QG ſupra QN ſit NG, arcus QN ſupra Qm ſit mN, & arcus Qm ſupra QF ſit mF; ſitque NG = Nm = mF, erunt

erunt in proportione arithmetica continua arcus QG : QN = QN : Qm = Qm : QF, & eodem modo erunt in proportione arithmetica arcus QR, QN, Qp, QS, quia etiam exceſſus NR = pN = Sp.

Corollarium IX.

Si linea KG (Fig. 18. n. 2.) tangit circulum, eſt arcus ωKF duplus AG, uterque enim arcus metitur angulum ωCG; ſed eſt KO duplus Ap; ergo eſt ωK + FO duplus pG; ſed pG = QZ, cuius eſt duplus QF; ergo Kω + FO = QC + FC; ſed Kω = QC; ergo FO = FC. Linea igitur AFy ſecat bifáriam arcum OC, cumque ſit integer OC = SN, duplus pN, erit arcus CF = FO = Sp = pN.

Corollarium X.

Cum in Figura 18. n. 3. ſit arcus Sp = pN per Corollarium IV., & cum, ob triangula, quæ ductis chordis pN, NR, Nm, NG reſultant æquilatera ex ipſa conſtructione Figuræ, & æqualia, ſit etiam arcus NG = pN, patet arcum SG triplum eſſe arcus NG; cumque ſit ſemiperipheria ZAG ad arcum AN, ut arcus SG ad arcum pN, erit etiam ZAG — SG : AN — pN = ZAG : AN, hoc eſt erit ZS : Ap = ZAG : AN; eſt ergo ZS triplus Ap, & quia Ap = QZ, eſt SZ triplus QZ.

Corollarium XI.

Per Corollarium V. eſt arcus QF duplus Cm, duplus QZ, duplus Ap per Corollarium III., ac proinde eſt QF = Ag; cumque ſit AN = QC, erit gN = FC = AS, & Sp = Fm = mN = RN, hoc eſt totus arcus FR triplus RN, & æqualis SG. Quia igitur totus circulus AZEG eſt triplus EN, & arcus SG eſt triplus GN, erit etiam, his arcubus demptis a primis, reſiduus SZEG triplus GE; ſed per Corollarium VI. etiam arcus GAZy eſt triplus GE; ergo arcus GAZy = SZEG, &, dempto utrinque arcu SZy, eſt arcus GS = Gy.

Corollarium XII.

Quia triangulum QRG eſt iſoſceles, angulus QRG = QGR; ergo arcus QCi = QAl; ſed arcus QG = QR; ergo arcus Gl = Ri; eſt autem Gl = gN = FC; nam, quia Gl eſt parallela AE, arcus Al = GE = AZ; ſed AQ = AN; ergo QZ = Nl = pg; ſed pN = NG; ergo gN = lG = Ri.

Corollarium XIII.

Cum chorda arcus FC ſit æqualis chordæ arcus Ri, patet eſſe totam GR=GF; nam, ſi intelligantur ductæ

rectæ

rectæ Ai, & RF, erit triangulum ARi = AFC, & eidem æquiangulum; ergo erit angulus ARi = AFC; ſed, ob triangulum iſoſceles ARF, etiam angulus AFR = ARF; ergo reſiduus FRG æqualis reſiduo RFG; ergo triangulum RGF, quod habet angulos ad baſim æquales, eſt iſoſceles, & latus GR = GF; adeoque Gi æqualis radio GC.

Corollarium XIV.

Si ducantur lineæ Al, AZ, El, EZ, Zl, Xi, quia arcus XCi = ZAl, erit triangulum ZEl iſoſceles ſimile triangulo ARl; quare erit Zl : lE = Al : Rl.

Corollarium XV.

Patet lineam Zl perpendicularem eſſe diametro AE; eſt enim AE parallela RG; quare angulus externus ZVE = ZlG recto, utpote in ſemicirculo. Eſt igitur etiam parallelogrammum iV rectangulum.

Corollarium XVI.

Quia triangulum ZAl eſt iſoſceles, erit rectangulum ZVl + AV² = Al² = EA × $\overline{AC - CV}$; & quia triangulum ZEl eſt iſoſceles, erit rectangulum ZVl + EV² = El² = EA × $\overline{AC + CV}$; ergo erit El² — Al² = EA

$EA \times AC + EA \times CV - EA \times AC + EA \times CV = EA \times 2CV$; ſed $2CV = GI$; ergo $EI^2 - AI^2 = EA \times GI$, ſeu $GZ \times GI$. Eſt autem etiam differentia quadratorum chordarum AI, & EI æqualis rectangulo ex ſumma laterum, ſeu chordarum in earundem differentiam; ergo huiuſmodi rectangulum eſt æquale rectangulo $GZ \times GI$.

Corollarium XVII.

Sed, quia ſumma quadratorum chordarum AI, & IE ſemper eſt æqualis quadrato AE, ob triangulum AIE rectangulum; Et, data ſumma duarum quantitatum, quantitas maior eſt æqualis dimidio ſummæ aucto ſemidifferentia, quantitas minor eſt æqualis dimidio ſummæ eadem ſemidifferentia diminuto; ergo erit $EI^2 = \frac{AE^2}{2} + GZ \times CV$, & $AI^2 = \frac{AE^2}{2} - GZ \times CV$. Eſt autem $\frac{AE^2}{2}$ æquale quadrato lateris quadrati circulo AGEZ inſcripti; ergo quadratum huius lateris auctum rectangulo $GZ \times CV = EI^2$, & eiuſdem lateris quadratum eodem rectangulo diminutum æquale AI^2.

Corollarium XVIII.

Cum dictum fuerit Corollario XIV. radium RI eſſe ad chordam AI, ut chorda IE ad chordam ZI, patet eſſe

eſſe radium ad chordam cuiuslibet arcus AI, ut chorda arcus, qui cum arcu AI eſt complementum ad ſemicircumferentiam ad chordam arcus dupli ipſius AI; vel etiam patet eſſe ſinum rectum arcus 30.° ad ſinum rectum arcus $\frac{1}{2}$ AI, ut ſinus rectus arcus, qui cum arcu $\frac{1}{2}$ AI eſt complementum ad quadrantem, ad ſinum rectum arcus AI.

Corollarium XIX.

Quia arcus $SZ = Zy$, ſuppoſitis chordis ZS, & Zy, triangulum SZy erit iſoſceles; ergo erit $SZ^2 = GZu$; ergo erit $GZ : SZ = SZ : Zu$.

Corollarium XX.

Cum ſit arcus $SZ = Zy = XF$, patet etiam arcum SG eſſe æqualem $Gy = RF$; erit igitur triangulum RAF ſimile triangulo SZy; eſt ergo $AF : RF = SZ : Sy$. Hinc iterum conſtat, quod dixi Corollario XVIII.

Corollarium XXI.

Demonſtratum fuit eſſe arcum $AS = GI$; ergo linea SG eſt parallela AI, & triangulum SRG eſt ſimile triangulo ARI; ergo eſt $SG : AI = RG : RI$, & $SG \times RI = AI \times RG$. Eſt autem per Corollarium XVII. $AI^2 = \frac{AE^2}{2} - GZ \times CV$; adeoq; $AI = \sqrt{\frac{AE^2}{2} - GZ \times CV}$,

ac

ac proinde $RG \times AI = RG \times \sqrt{\frac{AE^2}{2} - GZ \times CV}$.

Poteſt vero denominari $AE = 2a$, $SG = RF$, quæ eſt ſubmultiplex $AE = ma$, $CV = \frac{GI}{2} = \frac{Ri}{2} = x$; Cumque fuerit demonſtratum Corollario XIII. eſſe $RI = Gi = a$; erit $RG = a + 2x$, & $GZ = AE = 2a$. Quo poſito habebitur $SG \times RI = ma \times a$, & $RG \times AI = \overline{a + 2x} \times \sqrt{2a^2 - 2ax}$, ac propterea $ma \times a = \overline{a + 2x} \times \sqrt{2a^2 - 2ax}$; $ma^2 = \sqrt{2a^4 + 6a^3x - 8ax^3}$, & $m^2a^4 = 2a^4 + 6a^3x - 8ax^3$, & tandem $x^3 - \frac{3a^2x}{4} + \frac{m^2a^3}{8} - \frac{a^3}{4} = o$.

Scholion.

In Figura potiſſimum 18. n. 3. plures omitto lineas, quas ſuppono ductas, ne nimia linearum copia confuſionem pariat.

Problema I.

Datis poſitione duobus circulis æqualibus KRC, ANE, (Fig. 18. n. 1.) & diametro QAK; item, data quacumque linea Kg ſecante circumferentiam circuli ANE in G, & g, invenire chordam KR arcus RK circuli KRC.

Reſo-

Reſolutio.

Conſtruatur Figura eo modo, quo dixi Theoremate IV., & eius Corollario I., & dicatur $QK = GZ = QH = 2a$; $Kg = ma$; utpote multiplex, vel ſubmultiplex QK; $KG = \frac{2a}{m}$; cum ſit $Kg : QK = KA : KG$; hoc eſt $ma : 2a = a : \frac{2a}{m}$; $KH = 2KR = 2x$, erit $GH = KH - KG = 2x - \frac{2a}{m}$, & $gH = KH - Kg = 2x - ma$. Hoc poſito erit $QR^2 = QK^2 - KR^2 = 4a^2 - x^2$; &, ob triangulum iſoſceles KQH, erit $Qg^2 = QH^2 - Kg \times gH = QK^2 - Kg \times gH = 4a^2 - 2max + m^2a^2$; $QG^2 = QH^2 - KG \times GH = QK^2 - KG \times GH = 4a^2 - \frac{4ax}{m} + \frac{4a^2}{m^2}$; & denique $GZ^2 = QK^2 = 4a^2$; ſed, ob triangula ſimilia QRg, & QGZ, eſt $QR : Qg = QG : GZ$; adeoque $QR^2 : Qg^2 = QG^2 : GZ^2$; ergo

$$4a^2 - x^2 : 4a^2 - 2max + m^2a^2 = 4a^2 - \frac{4ax}{m} + \frac{4a^2}{m^2} : 4a^2,$$

&, extremis, ac mediis in ſe invicem ductis,

$$16a^4 - 4a^2x^2 = 8a^2x^2 - 12ma^3x + 20a^4 - \frac{24a^3x}{m} + 4m^2a^4 + \frac{16a^4}{m^2},$$

 hoc

hoc eſt,
$$12a^2x^2 - 12ma^3x + 4a^4 - \frac{24a^3x}{m} + 4m^2a^4 + \frac{16a^4}{m^2} = 0,$$

&, diviſione peracta per $12a^2$,
$$x^2 - max + \frac{a^2}{3} - \frac{2ax}{m} + \frac{m^2a^2}{3} + \frac{4a^2}{3m^2} = 0,$$

&, terminis translatis,
$$x^2 - max - \frac{2ax}{m} = -\frac{a^2}{3} - \frac{m^2a^2}{3} - \frac{4a^2}{3m^2},$$

& denique, extracta radice, terminiſque ad ſimpliciorem expreſſionem redactis,
$$x = \frac{ma}{2} + \frac{a}{m} \pm \sqrt{\frac{8m^2a^2 - m^4a^2 - 4a^2}{12m^2}}$$

Problema II.

Datis poſitione duobus circulis æqualibus KRC, ANE (Fig. 19.), & diametro QAK, a puncto K ad circumferentiam concavam circuli ANE ducere lineam KD, quæ ita ſecta ſit in T, ut ſit tota KD ad partem KT, ut eadem KT ad reſiduum TD.

Reſo-

Reſolutio.

Sit $QK = 2a$, $KD = x$; Cum ſit $KD : QK = KA : KT$, hoc eſt $x : 2a = a : KT$, erit $KT = \frac{2a^2}{x}$. Erit ergo per conditionem Problematis $x : \frac{2a^2}{x} = \frac{2a^2}{x} : x - \frac{2a^2}{x}$, & $x^2 - 2a^2 = \frac{4a^4}{x^2}$, $x^4 - 2a^2x^2 = 4a^4$, $x^2 - a^2 = a^2\sqrt{5}$, & $x = \sqrt{a^2 + a^2\sqrt{5}}$.

Scholion.

Eodem modo reſolvi poterunt alia ſimilia problemata: Ex. gr. inveniri poterit linea KD, quæ ſit ad chordam TD, ut eadem chorda TD ad partem KT; item linea KD, quæ ſit ad partem KT in data proportione. Notandum tamen eſt datam proportionem eſſe non poſſe maiorem ea, quam habet $\sqrt{3} + 1 : \sqrt{3} - 1$ ob ea, quæ dixi Scholio II. Theorematis III.

THEOREMA V.

Dato circulo RiX (Fig. 20.), ſi in hoc ducetur quælibet chorda Ri, atque in hac, ſumpta qualibet parte RI, fiat Gi = RI, dico, quod ſi per puncta I, & G deſcribetur circulus GIZ dato æqualis, & inſuper ad-

dantur a puncto G per centrum GZ, & a punctis X ad G, atque R ad Z, XG, RZ, erit ZG parallela XR, & XRZG parallelogrammum.

Demonſtratio.

Quia $Rl = Gi$ per conſtructionem, addita li, eſt $Ri = Gl$; ergo arcus $Gl = Ri$; ſed totus arcus $XiR = GlZ$; ergo reſiduus $Xi = Zl$; ergo angulus $XRG = ZGR$; cumque hi anguli ſint alterni, & æquales, ZG erit parallela XR; ſed $XR = ZG$, quia ſunt diametri circulorum æqualium; ergo XRZG eſt parallelogrammum. Q. e. d.

Corollarium I.

Ductis a punctis X, & Z ad puncta l, & i rectis Xl, Xi, & Zl, Zi, etiam XlZi erit parallelogrammum; nam eſt $Xi = Zl$, & illa huic parallela, cum anguli Xil, ilZ alterni, utpote in ſemicirculo, ſint recti.

Corollarium II.

Quia $XR^2 + RZ^2 + ZG^2 + XG^2 = XZ^2 + RG^2$, & $Xl^2 + lZ^2 + Xi^2 + Zi^2 = XZ^2 + li^2$, (Inſtitut. Geom. P. Ab. Grandi par. 1. Coroll. 1. prop. 23.), erit prima ſumma ad ſecundam, ut $XZ^2 + RG^2 : XZ^2 + li^2$.

Corol-

Corollarium III.

Si a centris A, & C ad puncta N, & Q ducantur radii AN, CN, CQ, AQ, erit Figura ANCQ parallelogrammum; cumque AC, & QN ſint diagonales, ſe mutuo bifariam ſecabunt in O; & quia AR eſt æqualis, & parallela CG, triangula AOR, GOC erunt ſimilia, & æqualia; ac proinde etiam RG erit bifariam ſecta in O; cum igitur ſit RO = GO, & Rl = Gi per conſtructionem erit lO = iO; quare lineæ RG, XZ, AC, NQ, li erunt omnes bifariam ſectæ in eodem puncto O. Eſt autem per proprietatem circuli RO × Oi = QO × ON = ON²; ergo RO : ON = ON : Oi = ON : lO.

Corollarium IV.

Cum ſit AC² + QN² = 4CN², ſitque AC² = RZ², patet RZ² + QN² æqualia eſſe quadrato ZG diametri.

Corollarium V.

Me tacente clarum eſt etiam arcum XQ æqualem eſſe NZ, QG = RN, Qi = lN, Ni = lQ, XN = QZ, eorumque arcuum æqualium ſubtenſas eſſe inter ſe parallelas, & æquales, quarum extrema lineis rectis connexa

nexa efficiunt Quadrilatera, quæ omnia ſunt parallelogramma.

Corollarium VI.

Quia demonſtratum fuit $ON^2 = RO \times Ol$, erit $QN^2 = 4RO \times lO$; ſed $4RO \times lO = RG \times li$; ergo $RG \times li = QN^2$. Patet igitur communem chordam QN utriuſque arcus æqualis QlN, QiN eſſe mediam geometricam inter totam RG, & a ſupradictis arcubus ſectam li.

Corollarium VII.

Cum linea AC ſit perpendicularis QN, erit quadratum radii AN æquale quadrato AO, & quadrato ON; ſed $ON^2 = RO \times Ol$; ergo $AN^2 = AO^2 + RO \times Ol$. Ducta autem Am perpendicularis rectæ RG, erit AN^2, ſeu $Ai^2 = AO^2 + Oi^2 + 2iO \times Om$; ergo $AO^2 + Oi^2 + 2iO \times Om = AO^2 + RO \times Ol$, &, dempto utrinque AO^2, $Oi^2 + 2iO \times Om = RO \times Ol$; ſed $RO \times Ol = Rl \times lO + lO^2$, & $lO^2 = iO^2$; ergo, his demptis, $Rl \times lO = 2iO \times Om$, duplus $iO \times Om$; cumque ſit $iO = lO$; ergo Rl eſt dupla mO; ſed eſt etiam GR dupla RO; ergo Gl eſt dupla Rm, vel mi.

Corol-

Corollarium VIII.

Cum ſit GR dupla RO, Ri dupla Rm, & Rl dupla mO, erit GR : RO = Ri : Rm = Rl : mO; ergo GR × Rm = RO × Ri, GR × mO = RO × Rl, & Ri × mO = Rm × Rl.

Corollarium IX.

Quia Rl aucta lm efficit Rm dimidium ſummæ rectæ Ri, & li eadem lm diminuta pariter efficit dimidium ſummæ im, erit lm ſemidifferentia inter Rl, & li. Linea vero Ri erit arithmetice diviſa in l, & m, cum ſit li + Rl = 2 Rm, adeoque li : Rm = Rm : Rl.

Corollarium X.

Cum angulus SZG ſit æqualis qCG, ſitque primus ad circumferentiam, ſecundus ad centrum erit arcus GS duplus qG; erit ergo Gq = qS; ſed etiam Qq = qN; ergo reſiduus QG = NS = RN. Eſt igitur triangulum RNS iſoſceles, & rectangulum RZS + $NS^2 = NZ^2$.

Corollarium XI.

Ducta perpendiculari Amb in lineam Ri, erit triangulum AmO ſimile triangulo RlZ; ergo angulus

RZl

RZl erit æqualis bAt, & arcus lS duplus bt. Cum autem ſit angulus AOr rectus, & Om perpendicularis Ar, erit $Ar : AO = AO : Am$, & $AO^2 = Ar \times Am$; ſed, ob triangulum iſoſceles RAi, rectangulum $RO \times Oi + AO^2 = Ai^2$; ergo rectangulum $RO \times Oi + Ar \times Am = Ai^2$, ſeu, quia $RO \times Oi = ON^2$; $ON^2 + rA \times Am = Ai^2$.

Corollarium XII.

Sit modo linea RG (Fig. 21.) ſecta in tres partes æquales, in l, & i, ſi a punctis Z, & X ad l, & i ducentur lineæ Zl, & Xi, hæ productæ tranſibunt per centra A, & C; ſecant enim bifariam, & ad angulos rectos chordas Ri, & Gl; quare puncta m, & l Figuræ ſuperioris concurrent in unum.

Corollarium XIII.

Per Corollarium VI. $RG \times li = QN^2$; ſed $li = Rl$ per conſtructionem; ergo etiam $RG \times Rl = QN^2$; ſed huiuſmodi rectangulum eſt æquale quadrato tangentis ductæ a puncto R ad circulum GNZ; ergo ſi a puncto R ad n ducatur $Rn = QN$, illa erit circuli GNZ tangens.

Corol-

Corollarium XIV.

Per Corollarium VIII. $GR \times Rm = RO \times Ri$; ſed $GR \times Rm$, ſeu $GR \times Rl = QN^2$; ergo $QN^2 = RO \times Ri$, & $RO : QN = QN : Ri$. Item $GR \times mO = RO \times Rl$; ſed $GR \times mO = \frac{GR \times Rl}{2} = \frac{QN^2}{2}$; ergo $\frac{QN^2}{2} = RO \times Rl$, & $RO : \frac{QN}{\sqrt{2}} = \frac{QN}{\sqrt{2}} : Rl$. Pariter $Ri \times mO = Rm \times Rl$; ſed $Ri \times mO = \frac{Ri \times RO}{3} = \frac{QN^2}{3}$; ergo $\frac{QN^2}{3} = Rm \times Rl$; ſed $Rm \times Rl = Rl^2$; ergo $\frac{QN}{3} : Rl = Rl : QN$.

Corollarium XV.

Quia $Rl = li$, & Zl eſt perpendicularis Ri, erit $RZ = Zi$, adeoque triangulum RZi iſoſceles; ergo erit $RG \times Gi + Zi^2 = ZG^2$. Sed, ducta Ca perpendiculari Zl, & addita iV, eadem de cauſſa etiam triangulum AiV eſt iſoſceles; ergo $AZ \times ZV + Vi^2 = Zi^2$; ergo $RG \times Gi + AZ \times ZV + Vi^2 = ZG^2$, &, dempto utrinque Vi^2, quod eſt æquale quadrato radii, $RG \times Gi + AZ \times ZV = \frac{3ZG^2}{4}$.

Corollarium XVI.

Rectangulum $RG \times Gi + Zi^2 = ZG^2$, & rectangulum $RO \times Oi + ZO^2 = Zi^2$; ergo $RG \times Gi + RO \times Oi + ZO^2 = ZG^2$. Eſt autem $RG \times Gi = 4RO \times Oi = 4ON^2$; ergo $RG \times Gi + RO \times Oi = 5RO \times Oi = 5ON^2$, & $5ON^2 + ZO^2 = ZG^2$; ſed $ZG^2 = RZ^2$, ſeu $Zi^2 + 4ON^2$; ergo $5ON^2 + ZO^2 = 4ON^2 + Zi^2$; ergo exceſſus quadrati Zi ſupra quadratum ZO eſt æquale quadrato ON.

Corollarium XVII.

Si exceſſus quadrati Zi, ſeu RZ ſupra quadratum ZO eſt æquale quadrato ON, erunt $4RZ^2 = 4ZO^2 + 4ON^2 = XZ^2 + QN^2$; ſed, ſi intelligantur ductæ chordæ ZN, XN, XQ, ZQ, etiam $XN^2 + ZN^2 + XQ^2 + ZQ^2 = XZ^2 + QN^2 = 4RZ^2$; ergo ſumma ſupradictorum quadratorum eſt quadrupla quadrati RZ.

Corollarium XVIII.

Quia $XZ^2 + QN^2 = 4RZ^2 = 4RZ \times ZS + 4ZR \times RS$, & $QN^2 = ZR \times RS$; ergo erit $XZ^2 = 4RZ \times ZS + 3RZ \times RS$.

Corol-

Corollarium XIX.

Sit Rl = Gi (Fig. 22.) æqualis radio AR, ducta Zl, & huic perpendiculari CVa, & addita ai, item iC, Ra, & lC, tria quadrilatera iGCa, aiCZ, RlCa erunt parallelogramma, & erunt æqualia; Eſt enim Gi æqualis, & parallela aC, adeoque ai æqualis, & parallela CG; ſed CG = CZ; ergo CZ æqualis, & parallela ai; quare aZ æqualis, & parallela Ci; cumque ſit aC parallela, & æqualis Rl, erit Ra parallela, & æqualis lC. Sunt autem tria parallelogramma aG, iZ, CR dupla trianguli iaC, ſeu lCa; ergo ſunt inter ſe æqualia.

Corollarium XX.

Quia Ra = lC = CZ = ai, triangulum Rai eſt iſoſceles; ſed Rai eſt ſimile lpi, & apC; ergo etiam iſta triangula ſunt iſoſcelia; ergo eſt Ri : ai = ai : ap.

Corollarium XXI.

Cum, ducta Ai, ſit RAia rhombus; ſi a puncto A ad a ducatur Aa, hæc ſecabit bifariam, & ad angulos rectos lineam Ri; ergo erit Aa parallela Zl; ſed, quia CVa eſt perpendicularis Zl, erit perpendicularis etiam Aa. Cum igitur Aa efficiat angulum rectum cum radio Ca, erit tangens circuli GNZ in puncto a.

Corollarium XXII.

Eſt Aa dupla ma, & Zl dupla lV; ſed ma = lV, quia mV eſt parallelogrammum; ergo Aa eſt æqualis, & parallela lZ; ergo, ducta Al, ZA eſt parallelogrammum. Producta ergo linea Al in L, quia AL eſt parallela aZ, erit arcus al = ZL; ſed arcus al = aZ; ergo arcus lZL eſt triplus al.

Corollarium XXIII.

Cum etiam Ci ſit æqualis, & parallela Al erit CA parallelogrammum. Parallelogrammata autem CA, ZA ſuper latera trianguli ClZ æqualia erunt parallelogrammo Zi ſuper eiuſdem baſim; eſt enim ZA = Zy, CA = iZ, quæ ſunt ſuper eadem baſi, & inter eaſdem parallelas.

Corollarium XXIV.

Demonſtratum fuit Aa eſſe tangentem circuli GNZ in puncto a, & æqualem eſſe lZ; erit ergo rectangulum $LAl = Aa^2 = lZ^2$, hoc eſt, cum Al ſit æqualis aZ, erit linea compoſita ex chorda arcus dati, & chorda tertiæ eius partis ad chordam duarum partium tertiarum eiuſdem arcus, ut eadem chorda ad chordam tertiæ partis.

Corol-

Corollarium XXV.

Conſtat triangula omnia alu, lGz, uCz, aCZ &c. eſſe iſoſcelia, & ſimilia; erit ergo aC : aZ = al : au, hoc eſt, quia aZ = al, erit aC : al = al : au; ſed eſt inſuper al : au = au : uy; ergo quatuor lineæ Ca, al, au, uy ſunt continuo proportionales.

Corollarium XXVI.

Patet eſſe Al = lu = yz = zL; adeoque eſſe ly = uz, Ay = Lu, & li = Cu = Cz.

Corollarium XXVII.

Si in parallelogrammo AZ ducatur diagonalis ZA, erit $AZ^2 + al^2 = 2Aa^2 + 2Al^2$; ſed $Al^2 = al^2$; ergo, his demptis, $AZ^2 = 2Aa^2 + Al^2$; ſed $2Aa^2 = 2LAl$; ergo $AZ^2 = 2LAl + Al^2$. Eſt autem $AZ^2 = ZAn + AZn$, & $ZAn = LAl$; ergo $AZn = LAl + Al^2$; ergo differentia rectanguli AZn a rectangulo ZAn eſt æqualis Al^2.

Corollarium XXVIII.

Ob rhombum Ci, linea aG eſt perpendicularis lC, adeoque eſt perpendicularis AL; cum ergo ſit Ac = cL,

erit

erit triangulum AGL iſoſceles; ergo erit $LlA + Gl^2 = AG^2$; ſed, ob triangulum $AGl = Gzl$, & eidem æquiangulum, eſt arcus $ZL = lr = la$, ac proinde chorda $lr = lA$; ergo triangulum Alr eſt iſoſceles, eſtque $Gl^2 = AGr + lr^2$.

Corollarium XXIX.

Cum ſit $AG^2 = Am^2 + Gm^2 = LlA + Gl^2$, erit $Am^2 + Gm^2 = LlA + Gl^2$, cumque ſit $Gm^2 = ml^2 + Gl^2 + 2Glm$, erit $Am^2 + ml^2 + Gl^2 + 2Glm = LlA + Gl^2$, &, dempto utrinque Gl^2, $Am^2 + ml^2 + 2Glm = LlA$. Eſt autem $Am^2 + ml^2 = Al^2$; ergo $Al^2 + 2Glm = LlA$, & $2Glm = LlA - Al^2$; ſed $LlA = zAl = Al^2 + Alz$; ergo $2Glm = Al^2 + Alz - Al^2 = Alz$; ergo rectangulum Alz eſt duplum Glm.

Corollarium XXX.

Ducta zi, quia duo triangula liC, zCi ſunt æqualia, ob $li = Cz$, Ci commune, & angulum $liC = iCz$, erit $zi = Cl = ai$, adeoque ducta azH triangulum aiz erit iſoſceles, & æquale iaC; linea aH erit æqualis Ll, & perpendicularis GL; a puncto G ad H ducta GH, erit triangulum GzH æquale Glz; productis aC in D, & al in I arcus arcus lr, la, aZ, ZL, LH,

LH, GD, GI æquales; cumque ſit arcus lI = DH, ob angulum laI = DaH, erit etiam DH = lr — rI. Erit igitur totus circulus QZHG diviſus in octo partes, & deerunt duæ partes æquales arcui rI; ſi igitur arcus rI in quatuor partes æquales dividetur, erit arcus lr, dempta quarta parte arcus rI æqualis octavæ parti totius circuli. Verum eſt, quod dixi, etiamſi arcus ZL, Za, al &c. ſint ſinguli minores octava parte circuli, dummodo quarta pars arcus rI cuilibet ex dictis arcubus addatur.

Corollarium XXXI.

Dictum fuit Corollario XXIX. rectangulum Ala duplum eſſe Glm; ſed eſt Ala = luL = auD; ergo auD eſt duplus Glm. Patet etiam DuV eſſe æquale Glm, & DVa æquale eſſe Gml.

Corollarium XXXII.

Ob triangulum rectangulum lVu, eſt $lu^2 = lV^2 + Vu^2$; ſed eſt $lV^2 = DVa$; ergo $lu^2 = DVa + Vu^2$; ſed inſuper eſt $DVa + Vu^2 = DaV$; ergo $lu^2 = DaV$. Eſt autem per Corollarium XXV. $lu^2 = al^2 = Cau$; ergo $Cau = DaV$.

Corol-

Corollarium XXXIII.

Cum exceſſus ſemicircumferentiæ ſupra arcum GL ſit Z L, & exceſſus arcus G L ſupra GH, vel G l ſit LH = Z L, erunt arcus GHZ, GL, & GH arithmetice proportionales.

Corollarium XXXIV.

Quia triangulum AGL eſt iſoſceles, angulus ALG = LAG; ergo arcus Gl eſt duplus ox; ſed etiam arcus l a eſt duplus s x; ergo totus Ga eſt duplus os. Cum igitur ſit Gr + lr = 2ox, & Gl + la = 2os, erunt arithmetice proportionales Gr : ox = ox : lr, & Gl : os = os : la; cumque ſit differentia inter arcum Gr, & lr æqualis arcui rI, erit differentia inter arcum Gr, & ox = $\frac{rI}{2}$; & pariter, cum ſit differentia inter arcum Gl, & la æqualis arcui Gr, erit differentia inter Gl, & os = $\frac{Gr}{2}$.

Corollarium XXXV.

Data chorda R i, quæ ſemper erit pars diametri dati RX, nota erit RG, item RZ; inveniri ergo ſemper poterit, cuius arcus ſit chorda portio ZS lineæ RZ

ope

ope tabularum trigonometricarum; nam, ſi intelligatur ducta GS, hæc erit perpendicularis RZ in S, quia angulus GSZ eſſet in ſemicirculo; in triangulo igitur acutangulo RZG; $RG^2 = GZ^2 + RZ^2 - 2RZS$; adeoque $2RZS = GZ^2 + RZ^2 - RG^2$, & $ZS = \frac{GZ^2 + RZ^2 - RG^2}{2RZ}$. Quod ſi triangulum RZG haberet in Z angulum obtuſum, tunc RZ ſecaret ſemicircumferentiam GHZ, quare eſſet $RG^2 = GZ^2 + RZ^2 + 2RZS$, & $\frac{RG^2 - GZ^2 - RZ^2}{2RZ} = ZS$.

Corollarium XXXVI.

Dicatur modo diameter RX (Fig. 23.) circuli RQX = 2a, & fiat chorda Ri = $\frac{3RX}{4} = \frac{3a}{2}$: producatur Ri in G ita, ut ſit Gi = Ri = a, hoc eſt æqualis radio, & reliqua perficiantur, ut in ſuperiori Figura. Cum li ſit æqualis iy = Cz = $\frac{a}{2}$; erit Cz = zZ; quia ergo Corollario XXIII. dictum fuit parallelogrammum ZA = Zy, & CA = iz, eſtque Zy = iz, ob æqualem baſim, æqualemque altitudinem, erit etiam ZA = CA. Deinde, quia demonſtratum fuit Corollario XXV. eſſe Ca : al = al : au = au : uy, & eſt Ca = a,

$au = \frac{a}{2}$; erit $a : \frac{a}{\sqrt{2}} = a : \frac{a}{\sqrt{2}}$: $\frac{a}{2} = \frac{a}{2} : \frac{a}{2\sqrt{2}}$. Rurſus, quia eſt Ci : yu = Ca : au, & aZ : uz = aC : Cu, & eſt Ca : au = aC : Cu, erit Ci : yu = aZ : uz; ſed Ci = aZ; ergo yu = uz = ly. Eſt ergo AL ſeptupla, Az quintupla, Au quadrupla, Ay tripla, Al dupla uz. Cum igitur ſit $uz = \frac{a}{2\sqrt{2}}$, erit $AL = \frac{7a}{2\sqrt{2}}$, $Az = \frac{5a}{2\sqrt{2}}$, $Au = \frac{2a}{\sqrt{2}}$, $Ay = \frac{3a}{2\sqrt{2}}$, $Al = \frac{a}{\sqrt{2}}$; cumque ſit rectangulum $LAl = Aa^2 = lZ^2 = \frac{7a^2}{4}$, erit $Aa = lZ = \frac{a\sqrt{7}}{2}$; & pariter, cum ſit rectangulum $RG \times li = QN^2$ per Coroll. VI. $= \frac{5a^2}{4}$, erit $QN = \frac{a\sqrt{5}}{2}$. Quia tandem Au inventa fuit æqualis $\frac{2a}{\sqrt{2}} = a\sqrt{2}$, erit æqualis lateri quadrati circulo ZlG inſcribendi; &, quia la eſt dimidium Au, erit chorda la æqualis ſinui recto arcus 45°.

Corollarium XXXVII.

Per Coroll. XXVII. $AZ^2 = 2LAl + Al^2$; ſed per Coroll. præc. $LAl = \frac{7a^2}{4}$, & $2LAl = \frac{7a^2}{2}$, $Al^2 = \frac{a^2}{2}$; ergo $AZ^2 = \frac{8a^2}{2} = 4a^2$ æquale quadrato diametri,

adeo;

adeoque $AZ = ZG$; eſt igitur AZG triangulum iſoſceles. Eſt autem per idem Corollarium XXVII. rectangulum $AZn = LAl + Al^2 = \frac{7a^2}{4} + \frac{a^2}{2} = \frac{9a^2}{4} = Ri^2$; ergo $AZ : Ri = Ri : Zn$. Ex eo, quod AZ ſit æqualis diametro, patet circumferentiam RiX eandem biſecare; cumque lineæ AZ, al, ob parallelogrammum, ſe mutuo bifariam dividant, erit earum ſectio in puncto circumferentiæ RiX.

Corollarium XXXVIII.

Quoniam triangulum AZG præcedenti Corollario demonſtratum fuit iſoſceles, &, ſi ſupponatur ducta Zr, hæc evadit perpendicularis AG, quia angulus in ſemicirculo rectus, erit AG a circulo bifariam ſecta in r, quare erit rectangulum $GAr = AGr = LAl$; ſed duo rectangula GAr, & AGr ſunt æqualia quadrato AG; ergo quadratum AG eſt duplum LAl, ſeu Aa^2.

Corollarium XXXIX.

Sit iterum circulus RiX (Fig. 24.), ſitque chorda Ri æqualis lateri quadrati eidem circulo inſcripti. Conſtruatur Figura eodem modo, quo ſuperiores, & ſupponatur $RX = 2a$, erit $Ri = a\sqrt{2}$, & $li = a\sqrt{2} - a$,

$RG = a\sqrt{2} + a$, adeoque $RG \times Ii = \overline{a\sqrt{2} + a} \times \overline{a\sqrt{2} - a} = a^2$. Eſt autem per Coroll. VI. $RG \times Ii = QN^2$; ergo $QN^2 = a^2$; & $QN = a$, eſt æqualis radio; ergo arcus QxN eſt 60°.

Corollarium XL.

Cum in parallelogrammo RAia angulus RAi ſit rectus per conſtructionem, etiam reliqui erunt recti; quare radii aR, & ai erunt tangentes circuli RiX in R, & i. Linea ap, quæ eſt perpendicularis Ra, erit perpendicularis etiam parallelæ IC; &, quia per Coroll. XX. eſt $Ri : ai = ai : ap$, hoc eſt $a\sqrt{2} : a = a : \frac{a}{\sqrt{2}}$, erit Ri dupla ap, adeoque erit ap ſinus rectus arcus aI; ſed eſt tota $aI = GI = Ri$; ergo aI eſt dupla ap; ergo radius, qui ſecat bifariam, & ad angulos rectos chordam aI, bifariam ſecat etiam arcum aII. Et, quoniam etiam linea ArG debet ſecare arcum $Ir = Ia = Ii$, linea ArG tranſibit per extremum I lineæ aI, & nullus erit arcus rI; circulus ergo IZHG per Corol. XXX. erit diviſus in octo partes æquales.

Corollarium XLI.

Quia arcus $GHL = IZL$, erit triangulum GLI Iſosceles, & æquale $AGz = AGa$; ſimile vero trian-

gulis

gulis Aar, & Ary; erit ergo GL : Gl, hoc eſt AG : Aa = Aa : Ar = Ar : ry, nimirum erit chorda arcus 135° ad latus quadrati, ut latus quadrati ad eandem chordam diminutam chorda arcus 45°, ut hæc eadem chorda ad duplum differentiæ lateris quadrati a radio.

Corollarium XLII.

Quoniam $RZ^2 = Zl^2 + Rl^2 = 2a^2 + a^2 = 3a^2$; erit $RZ = a\sqrt{3}$ æqualis lateri trianguli æquilateri circulo ZlGH inſcribendi.

Corollarium XLIII.

Arcus os 67° 30′ erit triſectus; nam, quia angulus LlZ eſt æqualis xAs, erit ZL duplus sx; ſed ZL = Rs = si; ergo si duplus xs; ſed etiam ox eſt duplus xs; nam angulus externus GlL eſt æqualis duobus internis rGl, GAl, & eſt triplus rGl, adeoque GAl eſt duplus rGl; ergo lr = ox = is; ergo ox duplus sx, & oi = ix = xs. Dictum autem fuit Coroll. XXXIX, arcum QxN eſſe 60°; ergo arcus sN + Qo = 7° 30′, ſeu, quia Qr = sN, etiam arcus Qr + Qo = 7° 30′.

Corollarium XLIV.

Sit circulus Rfx (Fig. 25.) cujus diameter RX = 2a, & ponatur Ri æqualis lateri trianguli æquilateri = $a\sqrt{3}$,

erit

erit $RG = a\sqrt{3} + a$, & $li = a\sqrt{3} - a$, ac proinde $RG \times li = QN^2 = \overline{a\sqrt{3} + a} \times \overline{a\sqrt{3} - a} = 2a^2$; ergo erit $QN = a\sqrt{2}$, hoc eſt æqualis lateri quadrati; adeoque arcus QiN $90°$.

Corollarium XLV.

Ob rhombum RAia, $Ri^2 + Aa^2 = 4AR^2$; ſed $Ri^2 = 3AR^2$; ergo $Aa^2 = AR^2$, & $Aa = AR$. Cum itaque, quæ debet eſſe tangens circuli ZQG, ſit æqualis radio, tanget circulum ZQG in communi ſectione N utriuſque circuli; quare puncta a, & N concurrent in unum. Cum autem linea ducta a puncto a, ſive N ad i ſit æqualis radio, erit arcus Ni $60°$; ſed arcus QN eſt $90°$ per Coroll. præced.; ergo arcus Qi eſt $30°$; quare linea RG triſecat utrumque arcum QlN, QiN in l, & i.

Corollarium XLVI.

Ex Corollario V. manifeſtum eſt, quod, ſi intelligantur ductæ chordæ arcuum Qi, Ql, quadrilaterum inde reſultans erit parallelogrammum, adeoque erit $QN^2 + li^2 = 2Ni^2 + 2lN^2$; ſed QN^2 per Coroll. XLIV. eſt æquale duobus quadratis radii, ſeu $2Ni^2$; ergo $li^2 = 2lN^2$, hoc eſt quadratum lineæ li interce-

ptæ

ptæ inter utrumque arcum QIN, QiN eſt duplum quadrati chordæ IN 30°.

Corollarium XLVII.

Quia triangulum RIZ eſt rectangulum, & RI, ZI ſunt duo radii, patet lineam RZ æqualem eſſe QN, ſeu lateri quadrati.

Corollarium XLVIII.

Cum angulus ALG per conſtructionem ſit 60°; & triangulum GAL iſoſceles per Coroll. XXVIII., erit idem triangulum æquilaterum; ergo erit AG = GL = AL, adeoque linea AC ſecabit bifarium arcum Ir.

Corollarium XLIX.

Ex Coroll. XXXV. $ZS = \frac{RG^2 - GZ^2 - RZ^2}{2RZ} = \frac{a\sqrt{3} - a}{\sqrt{2}}$; ſed hæc eſt chorda arcus 30°; ergo ZS eſt æqualis chordæ arcus 30°; ſed ZL eſt arcus 30°; ergo ZS concurrit in L, & eſt tota RL = AL, ac proinde triangulum ALR eſt iſoſceles, & ſimile triangulo ARI. Eſt igitur AL : AR = AR : AI; ſed per Coroll. XXV. eſt AR : AI, ſeu Ca : aI = aI : au = au : uy; ergo eſt AL : AR = AR : AI = AI : au = au : uy.

Corol.

Corollarium L.

Fiat circulus R i X (Fig. 26.), cuius diameter $RX = 2a$, & chorda $Ri = \sqrt{\frac{5a^2}{2} - a\sqrt{\frac{5a^2}{4}}}$, hoc eſt æqualis lateri pentagoni, erit $RG \times li = \sqrt{\frac{5a^2}{2} - a\sqrt{\frac{5a^2}{4}}} + a \times \sqrt{\frac{5a^2}{2} - a\sqrt{\frac{5a^2}{4}}} - a = QN^2 = \frac{3a^2}{2} - a\sqrt{\frac{5a^2}{4}}$, & $QN = a\sqrt{\frac{3}{2} - \frac{1}{2}\sqrt{5}}$.

Corollarium LI.

Quia chorda R i eſt per conſtructionem latus pentagoni, erit arcus correſpondens 72°; ergo arcus X i, ſeu Z l 108°, adeoque Z L 54°, & G L 126°. Eſt autem chorda G L = G A, ob triangulum A G L iſoſceles; ergo etiam G A eſt chorda 126°. Sed eſt, ob l a = l r = Z l, arcus l r 54°; ergo arcus G r eſt 18°. Eſt igitur rectangulum A G r æquale rectangulo ſub chorda 126°, & chorda 18°; ſed huiuſmodi rectangulum cum quadrato chordæ l r arcus 54° eſt æquale quadrato G l, ob triangulum A l r iſoſceles, & eſt G l latus pentagoni; ergo rectangulum ex chorda 126° in chordam 18°

18° una cum quadrato chordæ 54° eſt æquale quadrato lateris pentagoni, ſive quadratis laterum hexagoni, & decagoni.

Corollarium LII.

Cum ſit arcus GI = Ir, hoc eſt uterque 54°, & arcus Gr, ſeu lI 18°, patet lineam AG triſecare arcum GI, & lineam aI arcum Ir.

Corollarium LIII.

Per ſe manifeſtum eſt arcum Gr eſſe vigeſimam partem, arcum Ir decimam; arcum Gl quintam, arcum denique Hr quartam partem totius circumferentiæ ZHGI.

Scholion I.

Etiam in Figuris ad hoc Theorema pertinentibus aliquæ omiſſæ fuerunt lineæ, ob rationem allatam in Scholio Theorematis IV. Et, ſi quæ deſunt lineæ in Figuris omnibus huius tractatus, hoc eidem cauſſæ adſcribatur.

Scholion II.

Hoc unice notandum eſt in omnibus Figuris huc uſque ad examen revocatis ſuppoſuiſſe chordam Ri maiorem radio Rl, angulum ARl acutum, ac proinde

circulos RNX, ZNG ſe mutuo ſecare. Verum, tum Theorema, tum fere omnia, quæ ab ipſo deduxi, ſubſiſtent, etiam ſuppoſita chorda Ri minori radio Rl, angulo ARl obtuſo, atque propterea neutrum circulum ſe mutuo ſecare, quemadmodum attendenti perſpicue conſtabit.

Problema III.

Data qualibet recta Al, & qualibet RX maiori Al, quæ erit diameter circuli deſcribendi, a circulo deſcripto diametro RX arcum ſecare ita, ut linea data Al ſit chorda tertiæ partis arcus ſecti.

Reſolutio.

Super data recta Al fiat triangulum ARl iſoſceles, cuius ſingula latera ſint æqualia dimidio lineæ datæ RX. Centro A radio AR deſcribatur circulus RiX, producatur linea Rl uſque ad circumferentiam circuli in i, fiatque Gi æqualis Rl, æqualis radio. Per l, & G deſcribatur circulus ZlG æqualis circulo RiX, & producatur Al in L, erit LZl arcus ſectus a circulo ZlGL, cuius diameter ZG æqualis RX. Erit autem linea la = lA, æqualis chordæ tertiæ partis arcus ſecti LZl. Demonſtratio habetur potiſſimum ex Corollario XXII. Theor. V.

Scho-

Scholion.

Cum latus trianguli æquilateri dividat totam circumferentiam circuli in tres partes æquales, linea data A I, quæ debet eſſe chorda tertiæ partis arcus a circulo ſecti, non poteſt eſſe maior latere trianguli æquilateri, ut per ſe notum eſt.

Problema IV.

Dati circuli ZLGX (Fig. 27.) peripheriam in quinquaginta quatuor partes æquales, quam proxime dividere.

Reſolutio.

Fiat ut duplus ſinus rectus quintæ partis circumferentiæ circuli dati, ſeu, ut chorda 144° ad triplum radium auctum dimidia radice quadrata quinque quadratorum eiuſdem radii, & diminutum latere pentagoni, ita radius ad quartam proportionalem, hoc eſt Claritatis gratia ſit circuli dati diameter ZG = 2a, erit duplus ſinus rectus quintæ partis circumferentiæ, ſeu chorda 144° æqualis $\sqrt{\frac{5a^2}{2} + a\sqrt{\frac{5a^2}{4}}} = a\sqrt{\frac{5}{2} + \frac{1}{2}\sqrt{5}}$,

& latus pentagoni æquale $\sqrt{\frac{5a^2}{2} - a\sqrt{\frac{5a^2}{4}}} =$

$a\sqrt{\frac{5}{2}-\frac{1}{2}\sqrt{5}}$; adeoque erit triplus radius auctus dimidia radice quadrata quinque quadratorum eiuſdem radii, & diminutus latere pentagoni æqualis $3a+\frac{a\sqrt{5}}{2}-a\sqrt{\frac{5}{2}-\frac{1}{2}\sqrt{5}}$. Fiat igitur $ZQ=a\sqrt{\frac{5}{2}+\frac{1}{2}\sqrt{5}}$, $ZP=3a+\frac{a\sqrt{5}}{2}-a\sqrt{\frac{5}{2}-\frac{1}{2}\sqrt{5}}$, & ducatur QP, atque ex puncto C, ubi $CZ=a$, hoc eſt æqualis radio, agatur CH parallela QP, erit ZH quarta proportionalis quæſita æqualis $\dfrac{3a^2+\frac{a^2\sqrt{5}}{2}-a^2\sqrt{\frac{5}{2}-\frac{1}{2}\sqrt{5}}}{a\sqrt{\frac{5}{2}+\frac{1}{2}\sqrt{5}}}$.

Circulo ex puncto Z applicetur $ZL=ZH$, & ex puncto G applicetur GI æqualis lateri pentagoni, erit arcus IL quam proxime quinquageſima quarta pars totius circumferentiæ.

Ducatur a puncto Z ad punctum I chorda ZI, & ſupponatur radius $CZ=a=10000000$. Hoc poſito erit chorda $ZL=ZH=\dfrac{3a^2+\frac{a^2\sqrt{5}}{2}-a^2\sqrt{\frac{5}{2}-\frac{1}{2}\sqrt{5}}}{a\sqrt{\frac{5}{2}+\frac{1}{2}\sqrt{5}}}$ $=15469446$, & $\frac{ZH}{2}=7734723$. Eſt autem hic

nu-

numerus in tabulis ſinuum ſinus rectus, quam proxime arcus 50° 40′; ergo eſt ZL chorda arcus 101° 20′ quam proxime; ſed, quia chorda GI eſt per conſtructionem chorda 72°, erit ZI chorda arcus 108°. Cum igitur ſit arcus ZLI 108°, & arcus ZL 101° 20′, erit, hoc ſubtracto a primo, arcus LI 6° 40′, adeoque erit quinquageſima quarta pars totius circumferentiæ ZLGX.

Problema V.

Circuli dati ZLGX (Fig. 28.) peripheriam in tercentum ſexaginta partes æquales quam proxime dividere.

Reſolutio.

In circulo ZLGX ducatur diameter ZG, & a puncto G ducatur ſubtenſa

$$GH = \frac{3a^2 + \frac{a^2}{2}\sqrt{5} - a^2\sqrt{\frac{5}{2} - \frac{1}{2}\sqrt{5}}}{a\sqrt{\frac{5}{2} + \frac{1}{2}\sqrt{5}}};$$

producatur GH in R, donec HR ſit æqualis radio GC, & ſumatur GI = GC = RH, erit tota RI = GH. Igitur per R, & I deſcribatur circulus RID æqualis ZLGX; a centro A per H ducatur AHL, & a puncto L ad X ducatur LX æqualis lateri trianguli æquilateri. Erit arcus XH quam proxime duorum graduum, quo biſariam diviſo, erit circumferentia circuli ZLGX in

in tercentum ſexaginta partes æquales diviſa.

Etenim ex dictis Problemate præcedente arcus GH eſt $101° 20'$; ergo arcus ZH, qui cum arcu GH efficit ſemicircumferentiam, eſt $78° 40'$; ſed per Corol. XXII. Theor. V. arcus HZ eſt duplus ZL; ergo ZL eſt $39° 20'$; ergo totus arcus HL eſt $118°$; ſed arcus LX eſt per conſtructionem $120°$; ergo arcus XH eſt $2°$.

Scholion.

Patet igitur ex dictis circulo dato inſcribi poſſe, & circumſcribi quam proxime eas omnes Figuras regulares, quas commemoravi Coroll. XI. Theor. I. Inſuper, cum per ſuperiora Problemata dividi poſſit circumferentia circuli in quinquaginta quatuor partes æquales, dividi poterit, ſumendo duplum arcum, in viginti ſeptem partes æquales, & quilibet arcus erit $13° 20'$. Cum autem detur arcus $12°$, hoc ſubtracto ab arcu $13° 20'$, habebitur arcus $1° 20'$; ſed præterea habetur arcus unius gradus; ergo, hoc ſubtracto ab arcu $1° 20'$, habebitur arcus $20'$, qui eſt tertia pars arcus unius gradus. Quare circulo inſcribi, & circumſcribi poterunt Figuræ omnes regulares, quarum latera ſubtendunt arcus conſtantes ex integris gradibus, & tertiis, ſextis, duodecimis &c. partibus gradus. Ex his autem hactenus expoſitis, audax mihi eſſe non videor, ſi con-

ſi concludam, Theoremata a me excogitata non eſſe iudicanda omnino inutilia, ex quibus talia ſaltem proxime conſequuntur; & Problema de Triſectione arcus geometrice tentatum non adeo eſſe negligendum, ut aliquis, noſtris etiam temporibus, imo plures adeſſe non debeant, qui in hac Doctrina promovenda ingenii ſui vires exerant.

THEOREMA VI.

Si ab extremis diametri AF (Fig. 29. & 30.) circuli ABEF ad quodlibet punctum D extra circulum ducantur duæ lineæ FD, AD ſecantes circumferentiam in duobus aliis punctis B, & E, erit quadratum diametri AF æquale rectangulis DF × FE, & DA × AB, ſi triangulum DAF habebit angulos ad A, & F acutos, & idem quadratum erit æquale differentiæ rectanguli DAB a rectangulo DFE, ſi, in eodem triangulo unus angulus F erit obtuſus.

Demonſtratio primæ Partis:

Ducantur lineæ AE, BF, (Fig. 29.) addatur BE, & a puncto D per communem ſectionem I linearum AE, BF agatur DI producta in H; hæc erit perpendicularis diametro AF: nam, quia anguli AED, FBD, utpote conſequentes angulis AEF, FBA rectis, ſunt recti,

recti, per quatuor puncta I, E, D, B poteſt tranſire circulus; ergo angulus EBD = EID; ſed EBD = AFE; ergo AFE = EID; ſed EID, & EIH ſunt æquales duobus rectis; ergo EIH, & HFE ſunt æquales duobus rectis; ergo per quatuor puncta H, F, E, I tranſire poteſt circulus; ergo anguli FEI, & FHI ſunt æquales duobus rectis; ſed FEI eſt rectus; ergo etiam FHI eſt rectus, adeoque DH perpendicularis AF. Per quatuor puncta igitur D, E, H, A, item D, F, H, B tranſire poterunt circuli, ac proinde DFE = AFH, DAB = FAH; ſed AFH + FAH = AF²; ergo DFE + DAB = AF². Q. e. pr.um.

Demonſtratio ſecundæ Partis.

Sit angulus AFD (Fig. 30.) obtuſus, & fiant omnia ut ſupra, quia anguli IED, IBD ſunt recti, per quatuor puncta I, E, B, D tranſire poteſt circulus; ergo angulus ABE = EFA = EID; ſed EFA, & EFH ſunt æquales duobus rectis; ergo etiam EIH, & EFH ſunt æquales duobus rectis; quare per quatuor puncta I, E, F, H tranſire poteſt circulus, adeoque etiam anguli IEF, & IHF ſunt æquales duobus rectis, ſed IEF eſt rectus; ergo etiam IHF eſt rectus. Poſſunt itaque tranſire circuli per quatuor puncta D, H, F, B, & per quatuor puncta D, H, E, A; ergo DAB = HAF; ſed

HAF

HAF = AF^2 + AFH, & AFH = DFE; ergo DAB = AF^2 + DFE, & DAB — DFE = AF^2. Q. e. ſec.^{um}.

Corollarium I.

Si triangulum AFD (Fig. 29.) ſit acutangulum; ductis FN, AN, erit FN^2 = AFH = DFE, & AN^2 = FAH = DAB. Erunt igitur ſimilia triangula DFN, & FNE, item AND, & ABN; quare erit DF : FN = ND : NE, AD : AN = ND : NB, adeoque FD × NE = FN × ND, & AD × NB = AN × ND.

Corollarium II.

Cum ſit AD × BN = AN × ND, & FD × NE = FN × ND, erit AD × BN : FD × NE = AN : FN, ac proinde AD × BN × FN = FD × NE × AN; ſed parallelepipeda æqualia reciprocant baſes, & altitudines; ergo erit AD : FD = NE × AN : FN × BN.

Corollarium III.

Suppoſito arcu FN æquali NB, cum ex Corollario præcedenti ſit DA : DF = EN × NA : FN × NB, hoc eſt = FN^2, erit etiam DA : DF = EN × NA : DF × FE, ac proinde DA × FE = EN × NA; quare DA : AN = NE : EF; ſed DA : AN = AN : AB; ergo AN : AB = NE : EF, & AN × EF = AB × NE,

hoc eſt rectangulum ſub chordis AN, FE æquale eſt rectangulo ſubtenſarum AB, EN.

Corollarium IV.

Si a puncto N in lineas DA, DF demittantur perpendiculares NS, & NR, erunt triangula ENR, FNH, FNA, AHN, BNS inter ſe ſimilia, ut patet, adeoque erit rectangulum $AF \times RN = FN \times NE$; $FA \times NS = AN \times NB$; $RN \times FN = EN \times FH$; $NS \times NA = NB \times AH$; $EN \times NS = RN \times NB$.

Corollarium V.

Per hanc propoſitionem eſt $AF : FE = DF : FH$; ſed, ob triangula ſimilia DFH, DRN, $DF : FH = DN : NR$; ergo $AF : FE = DN : RN$, & $AF \times RN = DN \times FE$; ſed per Corollarium præced. $AF \times RN = FN \times NE$; ergo $DN \times FE = FN \times NE$: Eodem modo demonſtrabitur $DN \times AB = AN \times NB$; quare erit $DN \times FE : DN \times AB = FN \times NE : AN \times NB$, hoc eſt $FE : AB = FN \times NE : AN \times NB$.

Corollarium VI.

Quia ex Corollario IV. $AF \times RN = FN \times NE$; & $AF \times NS = AN \times NB$, erit $AF \times RN : AF \times$

NS

NS = FN × NE : AN × NB, ſeu RN : NS = FN × NE : AN × NB; ſed ex Corollario V. etiam FE : AB = FN × NE : AN × NB; ergo FE : AB = RN : NS.

Corollarium VII.

Cum anguli ad H, R, & S ſint recti per quatuor puncta F, R, N, H, & H, N, S, A tranſire poſſunt circuli, adeoque rectangula ADS, HDN, & FDR ſunt æqualia; ſed etiam ADB = FDE; ergo reſiduum AD × SB = DF × ER, & AD : DF = ER : SB.

Corollarium VIII.

Ex Corollario VI. eſt FE : AB = RN : NS, & alternando FE : RN = AB : NS; ſed, ob triangula ſimilia, RN : ER = SB : NS; ergo FE : ER = AB × BS : NS^2. Eodem modo demonſtrabitur eſſe AB : BS = FE × ER : NR^2.

Corollarium IX.

Segmenta diametri AH, HF ſunt in ratione compoſita DA ad DF, & ſinus anguli ADH ad ſinum anguli HDF; ſunt autem ſinus horum angulorum perpendiculares NS, & NR; ergo ſunt in ratione compoſita DA ad DF, & NS ad NR; ſed per Corollarium II. eſt DA : DF = EN × NA : FN × NB; ergo erit

AH ad HF in ratione compoſita rectanguli $EN \times NA$ ad $FN \times NB$, & NS ad NR, ſive erit $AH : HF = EN \times NA \times NS : FN \times NB \times NR$: cumque ſit etiam $AH : HF = AH^2 : HN^2$, erunt etiam hæc quadrata, ut ſupradicta parallelepipeda.

Corollarium X.

Cum dictum fuerit Corollario VI. eſſe $FE : AB = RN : NS$, erit AH ad HF, ſive AH^2 ad HN^2, ut parallelepipedum ex tribus chordis EN, NA, AB ad aliud ex tribus ſubtenſis BN, NF, FE.

Corollarium XI.

Deſcriptis duobus circulis æqualibus FNA, & CNA (Fig. 31.) ſe mutuo ſecantibus in N, & A, ſi a ſectione N ducatur tangens PND circuli NCA, quæ ſecabit diametrum AF perpendiculariter in H, & ab extremo A diametri ad quodlibet punctum D ducatur AD ſecans circulos in B, & C, erit ſemper $PD : DN = BD : DC$; nam, ductis PB, AN, & NC, erit angulus $NPB = NAC = CND$; ergo erit PB parallela NC, adeoque $PD : DN = BD : DC$.

Corollarium XII.

Ob triangula ſimilia ABN, AND, & NCD erit $AB : BN = CN : CD$; ſed $BN = CN$; ergo $AB : BN = BN : CD$.

Corol.

Corollarium XIII.

Si producatur recta F N, donec ſecet circulum ASN (Fig. 32.) in C, & a puncto A per C ducatur recta ACD, erit arcus PF æqualis FN, æqualis NB; Etenim ducta A N, angulus A N F in ſemicirculo eſt rectus, adeoque rectus eſt etiam ANC; quare AC eſt diameter æqualis AF. Eſt igitur FAC triangulum iſoſceles, & quia AN eſt perpendicularis baſi FC, angulus FAN = CAN, & arcus NB = NF; ſed, quia PB eſt parallela FN, arcus NB = FP; ergo tres arcus PF, FN, NB ſunt æquales.

Corollarium XIV.

Hinc, ſi in circulo ABF ducta erit quælibet chorda AB, & ab extremo A per centrum G ducetur diameter AGF, facta AS = AB, & AC = AF, ductiſque rectis BSP, CF, erit arcus PNB triſectus, ut patet ex dictis.

Corollarium XV.

In Corollario XXIV. Theorematis V. demonſtravi eſſe lineam compoſitam ex chorda arcus dati, & chorda tertiæ partis arcus ad chordam duarum partium tertiarum eiuſdem arcus, ut eadem chorda duarum partium

tium tertiarum ad chordam tertiæ partis. Patet ergo in hac Figura Triſectionem arcus univerſalem pendere a geometrica ſolutione huius Problematis: Data qualibet chorda P B ſuper hanc conſtruere triangulum P N B, ſegmento BNP inſcriptum, ita ut ſit PB + BN : PN = PN : NB.

Scholion.

Hoc Theorema non differt a Propoſitione XXII. Partis primæ Inſtitutionum Geometricarum eximii Mathematici, & Geometræ acutiſſimi P. Ab. Grandi, niſi in hoc, quod ego circuli proprietati tribuo, quod ipſe ad triangula transfert.

THEOREMA VII.

Datis duobus circulis inæqualibus F N A, & S N L (Fig. 33. & 34.) ſe mutuo ſecantibus in N, & A, ductiſque a ſectionibus A, & N diametris AF, N T utriuſque circuli, erit rectangulum ſub diametro F A circuli maioris, & chorda S A minoris æquale rectangulo ſub diametro N T circuli minoris, & chorda N O maioris.

Demonſtratio.

Ducantur rectæ N A, & S T. Cum ſemicircumferentia F N A ſit ſimilis N S T, arcus F N ſimilis S N, ob

ob angulos æquales FAN, STN, & arcus AO ſimilis AT, ob eandem rationem; ergo, his demptis a ſemicircumferentiis, erit arcus NO ſimilis SA; ſed chordæ arcuum ſimilium ſunt, ut diametri; ergo FA: NT = NO: SA, ac propterea rectangulum FAS æquale TNO. Q. e. d.

Corollarium I.

Quia in triangulis FAN, TSN ſunt anguli NAF, NTS æquales, propter arcus ſimiles FN, SN, & ANF, TSN in ſemicirculo recti, triangula ſunt ſimilia; eſt ergo FA: AN = NT: ST; quare FA × ST = AN × NT; & arcus NQA, SAT ſunt ſimiles.

Corollarium II.

Cum ſit ſemicircumferentia FPA ſimilis NLT, & arcus AO ſimilis AT, erit arcus FPAO ſimilis NLTA, & ducta FO, erit FA: NT = FO: NA; ſed FA: NT = AN: ST per Coroll. præc.; ergo FO: NA = NA: ST.

Corollarium III.

Ex dictis arcus AO, AT ſunt ſimiles; ergo angulus AST = AFO, & linea ST parallela FO. Eſt ergo NS perpendicularis FO in H; ſed eſt AO perpendicularis FO; ergo eſt SHOr parallelogrammum.

Corol-

Corollarium IV.

Dictum fuit Coroll. II. eſſe $FO : NA = NA : ST$, adeoque $FO \times ST = NA^2$; ducta autem per centra linea PL, rectangulum $PXQ = MXL = \frac{NA^2}{4}$; ergo $FO \times ST =$ quadruplum rectanguli PXQ, ſeu MXL.

Corollarium V.

Si diametri FA, NT productæ concurrant in B, ut in Fig. 33., erit tota FB ad partem SB, ut quadratum chordæ FO ad quadratum chordæ NA; eſt enim $FB : SB = FO : ST$, ob parallelas; ſed per Coroll. II. eſt $FO : ST$ in ratione duplicata FO ad NA; ergo eſt $FB : SB = FO^2 : NA^2$. In hac eadem ratione erit etiam rectangulum NBO ad NBT, ut patet.

Corollarium VI.

Linea GR connectens centra circulorum FNA, SNL eſt media arithmetica inter ſubtenſas FN, AT ductas a punctis F ad N, & A ad T, ſi linea NA cadat inter G, & R, ut in Fig. 33.; & eadem GR æqualis eſt dimidio differentiæ ſubtenſæ FN a chorda AT, ſi linea NA cadat ultra centrum R, ut in Fig. 34. Eſt enim in primo caſu FN dupla GX, AT dupla XR;

ergo

ergo $FN + AT = 2GR$. In ſecundo autem caſu eſt FN dupla GX, AT dupla RX; ergo $FN - AT$ eſt dupla $GX - RX$, hoc eſt GR; ergo $RG = \frac{FN - AT}{2}$, &, ob parallelogrammum TANI, $= \frac{FI}{2}$. Hoc Corollarium verum eſt, etiamſi circuli ſe mutuo non ſecent; & rectæ FN, AT ductæ ab extremis utriuſque diametri ſint inter ſe parallelæ.

Corollarium VII.

Hinc, in Fig. 33., trapetium FATN æquale erit rectangulo $AN \times GR$; eſt enim æquale $\frac{FN + AT}{2} \times NA$. In Figura vero 34. erit triangulum $FAI = GR \times AN$; nam eſt $GR \times AN = \frac{FN - AT}{2} \times AN = \frac{FI}{2} \times NA$, quod productum eſt æquale triangulo FAI.

Corollarium VIII.

Cum ſit ſegmentum circuli FN ſimile SN, NO ſimile SA, AO ſimile AT, ſemicirculus denique FPA ſimilis ſemicirculo NZT, his demptis, erit trapetium FAON ſimile trapetio SATN, quæ proinde erunt, ut circuli, ſeu, ut quadrata diametrorum. Idem valet in Fig. 34.

THEOREMA VIII.

Si duo circuli æquales FNA, SNT (Fig. 35. & 36.) ſe mutuo ſecabunt in A, & N, ductis a ſectionibus A, & N per centra G, & R diametris AF, NT; item ducta per eadem centra linea GRZ, & chorda AOM per punctum O, ubi diameter NT ſecat arcum AO, occurrente rectæ GR in Z, erit GZ : AG = AG : AO = AO : OT, hoc eſt lineæ GZ, AG, AO, OT erunt continuo proportionales.

Demonſtratio.

Ducatur GO; quia FA eſt parallela, & æqualis NT, eſt etiam arcus FN = AO = AT, adeoque eſt chorda AT = AO = GR. Eſt igitur AOT iſoſceles, & ſimile OZR, AGO, AZG; ergo GZ : AG = AG : AO = AO : OT, Q. e. d.

Corollarium I.

Cum ſit angulus TAM = AGO, erit arcus TM duplus AO; ſed AO = AT; ergo TM duplus AT. Patet igitur diametrum NT triſecare arcum ATM.

Corollarium II.

Quia triangulum AOT eſt iſoſceles, erit $TRO + AO^2 = AR^2 = ZGR$; ſed $ZGR = GR^2 + GRZ$, & $GR^2 = AO^2$; ergo, his demptis, $GRZ = TRO = AZO$. Idem demonſtratur eodem fere modo in Fig. 36.

Corollarium III.

Hinc Problema iam ſupra ſolutum, nimirum dato circulo ANM, & qualibet recta non maiore latere trianguli æquilateri ab eodem abſcindere arcum ATM ita, ut recta data ſit chorda tertiæ partis eiuſdem arcus, reſolvi adhuc poterit hoc modo. Ducatur a centro R recta GR æqualis datæ; Supra GR fiat triangulum AGR iſoſceles, cuius latera AG, AR ſint æqualia radio circuli dati; centro G radio GA deſcribatur circulus ANF = ANM hunc ſecans in A, & N; a puncto N per centrum R ducatur NT, & a puncto A per O chorda AM. Dico factum. Demonſtratio habetur ex hoc Theoremate. Brevius hoc Problema reſolvetur, ſi fiet, ut radius RT ad lineam datam AT circulo applicatam ita eadem AT ad tertiam proportionalem TO, atque a puncto A per O ducetur AOM.

Corollarium IV.

Cum ex hoc Theoremate conſtet eſſe ZG, ſeu ZA : AG = AG : AO, erit rectangulum ZAO ſub recta ZA, & chorda tertiæ partis arcus ATM conſtanter æquale quadrato radii. Quod ſi ex puncto Z in baſim AG ſupponatur ducta perpendicularis, hæc ſecabit bifariam baſim, adeoque erit rectangulum ſub diametro FA, & dimidia baſi AG æquale ZAO. Arcus vero a puncto A ad punctum, ubi perpendicularis ſecat circulum FNA, erit 60°.

Problema VI.

Data qualibet linea AB (Fig. 37.) ſuper hanc conſtruere triangulum ABC ita, ut ſit ſumma lateris AB, & lateris minoris BC ad latus aliud AC, ut idem latus AC ad latus minus BC.

Reſolutio.

Fiat in A quilibet angulus acutus BAC, & in B angulus ABC anguli BAC duplus, ita tamen, ut anguli huiuſmodi ſimul ſumpti ſint minores duobus rectis, atque producantur latera AC, BC, donec concurrant in C. Dico factum.

De-

Demonſtratio.

Triangulo circumſcribatur circulus ACBD; quia angulus ABC per conſtructionem duplus eſt anguli CAB, erit arcus AC duplus arcus CB; ergo per Coroll. XXIV. Theor. V., erit AB + BC : AC = AC : BC. Q. e. d.

Corollarium.

Dato ergo quolibet triangulo ACB, cuius latera habeant proportionem in hoc Problemate aſſignatam, ſi huic circumſcribatur circulus ACBD, erit ſemper arcus ACB eiuſdem circuli triſectus.

THEOREMA IX.

SI in duobus circulis concentricis ANCF, IGLH (Fig. 38.) ducantur duæ diametri AC, HG ſe mutuo ſecantes ad angulos rectos, & per extrema A, & G diametrorum ducatur ſubtenſa AGF, erit quadratum diametri maioris AC duplum rectanguli ſub chorda FA circuli maioris, & intercepta AG inter utramque diametrum, & quadratum diametri minoris GH, erit duplum rectanguli ſub chorda GI circuli minoris, & eadem AG inter utramque diametrum.

Demonſtratio.

Ducatur linea CF; quia in triangulis AFC, ABG angulus AFC in ſemicirculo eſt rectus; & angulus ABG pariter rectus per conſtructionem, angulus vero FAC utrique triangulo eſt communis, erunt triangula ſimilia; ergo erit AC : AF = AG : AB, adeoque FAG = CAB; ſed rectangulum CAB eſt dimidium AC^2; ergo AC^2 eſt duplum FAG. Eſt autem etiam GH^2 duplum AGI; nam per Theor. VI. AGI = HGB; ſed HGB = $\frac{GH^2}{2}$; ergo GH^2 eſt duplum AGI. Q. e. d.

Corollarium I.

Erit ergo $AC^2 : GH^2$ = FAG : AGI; ſed FAG : AGI = FA : GI; ergo $AC^2 : GH^2$ = FA : GI.

Corollarium II.

Ductis BI, & AHE; quia angulus GBI eſt æqualis EAF, erit arcus GI ſimilis arcui CF, qui eſt dimidium arcus ECF; ergo erit AC : GH = CF : GI, & $AC^2 : GH^2 = CF^2 : GI^2$; ſed per Coroll. præc. $AC^2 : GH^2$ = FA : GI; ergo FA : GI = $CF^2 : GI^2$.

Corol-

Corollarium III.

Cum per Coroll. præced. ſit $FA : GI = CF^2 : GI^2$, erit $FA \times GI^2 = GI \times CF^2$, hoc eſt $FA \times GI = CF^2$. Eſt igitur rectangulum ſub chorda FA, & chorda GI æquale quadrato ſubtenſæ CF, & eſt $FA : CF = CF : GI$.

Corollarium IV.

Quoniam rectangulum CAB æquale eſt quadrato circulo ANCF inſcripto, & rectangulum HGB æquale eſt quadrato circulo minori IGLH inſcripto, etiam rectangula FAG, & AGI erunt dictis quadratis æqualia.

Corollarium V.

Ductis ergo lineis QG, QI; cum ſit QG latus quadrati circulo minori inſcripti, erit $AG : GQ = GQ : GI$; quare erunt triangula AGQ, GQI ſimilia: erit igitur rectangulum $AQG = AG \times QI$, hoc eſt rectangulum ex ſumma radii circuli majoris, & radii minoris in latus quadrati circulo minori inſcripti æquale eſt rectangulo ſub recta AG intercepta inter utramque diametrum, & chorda QI circuli minoris.

Corol-

Corollarium XI.

Si per centrum B ducatur MBD, & a puncto C per H GHN, ſitque MBD parallela rectis AF, & ICN, ductis ED, DC, EC, EF, GK, PK, PG, rectangula $\overline{ED + DC} \times DC$, $AC \times CX$, $\overline{PK + PG} \times PG$, & $FA \times GI$ erunt inter ſe æqualia; ſunt enim æqualia CE^2, ſeu CF^2, ſeu GK^2.

Scholion.

Notis diametris AC, RQ, patet ex Corollario V notam eſſe chordam QI, ac proinde etiam arcum QGI, a quo, ſi ſubtrahatur arcus QG conſtanter 90°, habebitur arcus GI, cui ſimilis eſt arcus EC, ſeu CF.

THEOREMA X.

Si fuerint duo circuli inæquales LFEM, IGHK, (Fig. 39.) ſed non concentrici, cujus diametri ſint ED, GH, atque ab extremis unius diametri E, & F per extrema alterius G, & H ducantur duæ rectæ FA, EA concurrentes in A, erit rectangulum ſub tota FA, & parte IL a ſectione I ad ſectionem L æquale rectangulo ſub AM, & HE poſita inter utramque diametrum.

Demonſtratio.

Ducantur chordæ EL, FM in circulo maiori; item HI, GK in circulo minori. Triangula AMF, AIH ſunt ſimilia, cum habeant angulum AMF æqualem AIH utrumque rectum, & angulum A communem; ergo $AM : AF = AI : AH$; ergo $AM \times AH = AF \times AI$; ſed per circuli proprietatem $AM \times AE = AF \times AL$; ergo ab his, demptis primis, erit $AM \times HE = AF \times IL$. Eodem modo demonſtrabitur $AE \times KM = AL \times GF$. Q. e. d.

Corollarium.

Rectangulum ſub linea IL inter utramque chordam HI, EL, & linea FG inter utramque diametrum GH, EF eſt æquale rectangulo ſub linea KM inter utramque chordam GK, FM, & linea HE inter utramque diametrum GH, FE; nam, ob triangula ſimilia FAM, GAK, eſt $FA : AM = AG : AK$, ac proinde $FA : AM = FG : KM$; ſed, ob triangula ſimilia FAM, ALE, eſt $FA : AM = AE : AL = FG : KM$, &, ob triangula ſimilia ALE, AIH, eſt $AE : AL = AH : AI = HE : IL$; ergo $HE : IL = FG : KM$, & $HE \times KM = IL \times FG$.

Scho-

Scholion.

Idem, quod dictum fuit ſupra, verum erit etiam, ſi duo circuli inæquales ſe mutuo ſecabunt, vel unus erit intra alium, ut duo circuli FMLE, GKIH, (Fig. 40.) & eodem modo demonſtrabitur, quemadmodum clarum eſt.

Problema VII.

Data qualibet linea LA, (Fig. 41.) quæ circulo poſſit applicari, & circulo LAMF ab eodem ſecare arcum FM ita, ut recta FE = LA ſit ſubtenſa duarum partium tertiarum eiuſdem arcus.

Reſolutio.

Circulo applicetur recta data LA; bifariam dividatur in B, & a puncto B per centrum ducatur BCE: addantur AE, LE, & fiat LF parallela AE: a puncto L per C ducatur LCM, & coniungantur FM, FE: dico factum.

Demonſtratio.

Cum ſit LF per conſtructionem parallela AE, chorda LA = FE, & triangula EOA, FOL ſunt iſoſcelia, adeoque tota EL = FA = AE; quare triangula LEA,

LEA, FAE ſunt iſoſcelia, & æqualia; ergo angulus LEA = LCB = FAE = ECM; ſed angulus FAE ad circumferentiam, & ECM ad centrum; ergo arcus FE duplus EM. Q. e. d.

Corollarium.

Cum ſit per conſtructionem FL parallela AE, & illa ſit perpendicularis FM, erit etiam AE eidem perpendicularis; quare erit $FA^2 + EM^2 = FE^2 + AM^2$; ſed per Coroll. XXIV. Theor. V. $FE^2 = \overline{FM + EM} \times EM$; ergo $FA^2 + EM^2 = FM \times EM + EM^2 + AM^2$, &, dempto utrinque EM^2, $FA^2 = FM \times EM + AM^2$.

Scholion.

Hunc ego Tractatum inſcripſi: *Theoremata ad Triſectionem arcus pertinentia*. Duplex autem me impulit cauſſa, ut hunc hiſce Theorematis titulum imponerem: Problema nempe Taurinenſe de Enneagono in circulo deſcribendo, quod ad hæc exaranda eſt adhortatum; & nonnullæ chordarum relationes, quæ ex iiſdem inferuntur, quæque, ob reſpectum, quem habent chordæ ad arcus, ad eorum perfectiorem cognitionem conducere fortaſſe nos poſſent: ex qua poſtea, etſi vix ſperandum, Problematis etiam ſolutio geometrica de triſectione arcus, vel etiam univerſalior de eiuſdem diviſione

ſione in data quacumque ratione poſſet prodire . Cæterum verius ipſum debuiſſem Tractatum inſcribere : *Theoremata de variis Cireulorum potiſſimum ſe mutuo ſecantium affectionibus* . Veritates autem , quæ ex mutua duorum circulorum ſectione oriuntur, ab iis profecto non erunt rejiciendæ , qui norunt , quid commodi attulerint Matheſi mutuæ curvarum ſectiones . Fateor tamen , me hanc vix delibaſſe materiem , ut eius potius ideam excitarem, quam, ut rite eandem perpenderem : eam nihilominus geometricis non carere proprietatibus non obſcure animadverti . Hinc ſpero alios acutiori ingenio præſtantes Geometras, atque tranquilliori fruentes otio , quam mihi ſit conceſſum , illud eſſe perfecturos , quod ego vix indicavi .

ERRATA			CORRIGE
Pag.	lin.		
7.	21.	ambituſi ſecund	ambitus ſecundi
9.	21.	$\frac{1d^3}{6}$	$\frac{ad^3}{6}$
19.	15.	corporls	corporis
42.	10.	$\frac{2BO^2}{3}$	$\frac{2BO^3}{3}$
62.	2.	æquiſita	acquiſita
72.	14.	circularum	circularem
77.	9.	$-\frac{m^3}{2r^3}$	$-\frac{m^3}{3r^3}$
80.	20.	aſſe	axe
126.	6.	ſunt	ſunt
137.	14.	$+a^2\sqrt{3}+Zy$	$+a^2\sqrt{3}+Zy^2$
142.	8.	InO	TnO
163.	4.	hoe eſt	hoc eſt
166.	15.	perpendicularis	perpendiculari
203.	5.	$=$ AZO	$=$ AOZ

Die 26. Aprilis 1757.

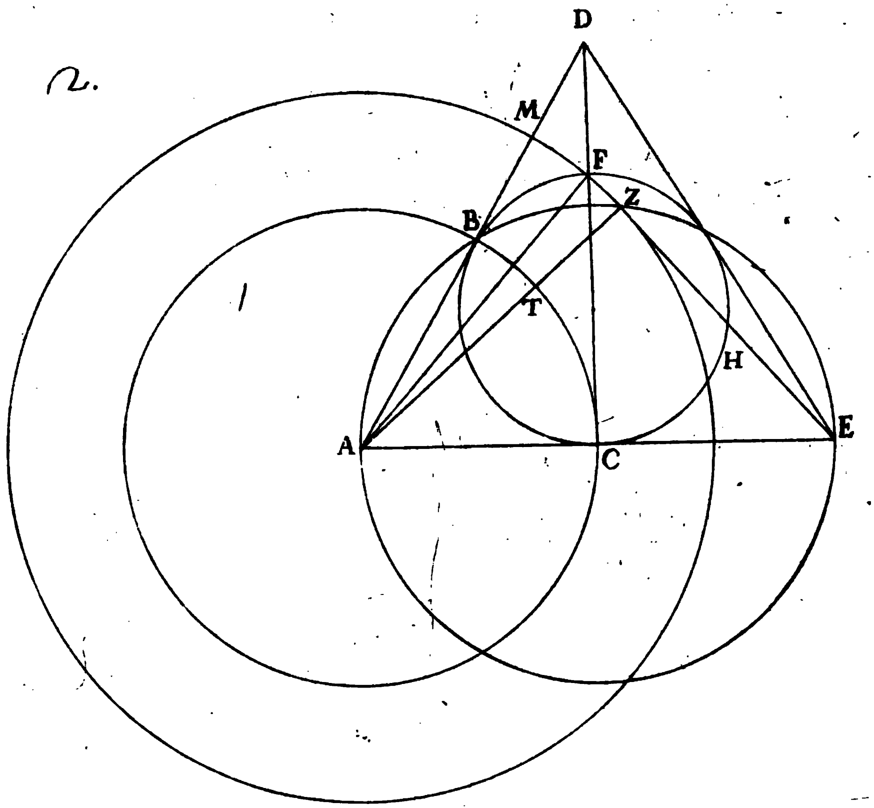
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F. Jo: Baptiſta Wabemans Ord. Præd. Sac. Theol. Magiſt. Vicarius Generalis S. Offitii Mediolani.

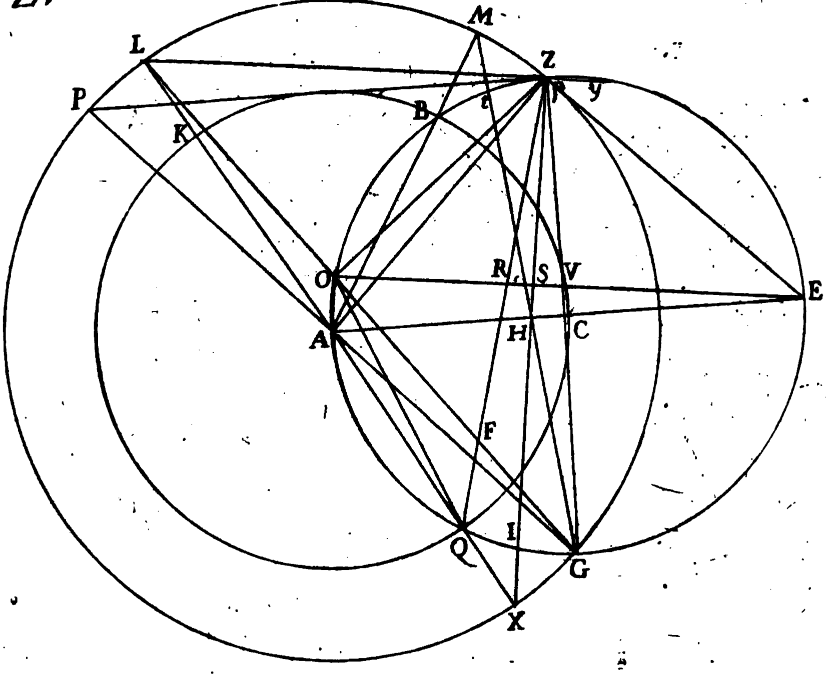
J. A. Viſmara pro Eminentiſs., & Reverendiſs. D. D. Cardinali Archiepiſcopo.

Vidit Julius Cæſar Berſanus pro Excellentiſſimo Senatu.

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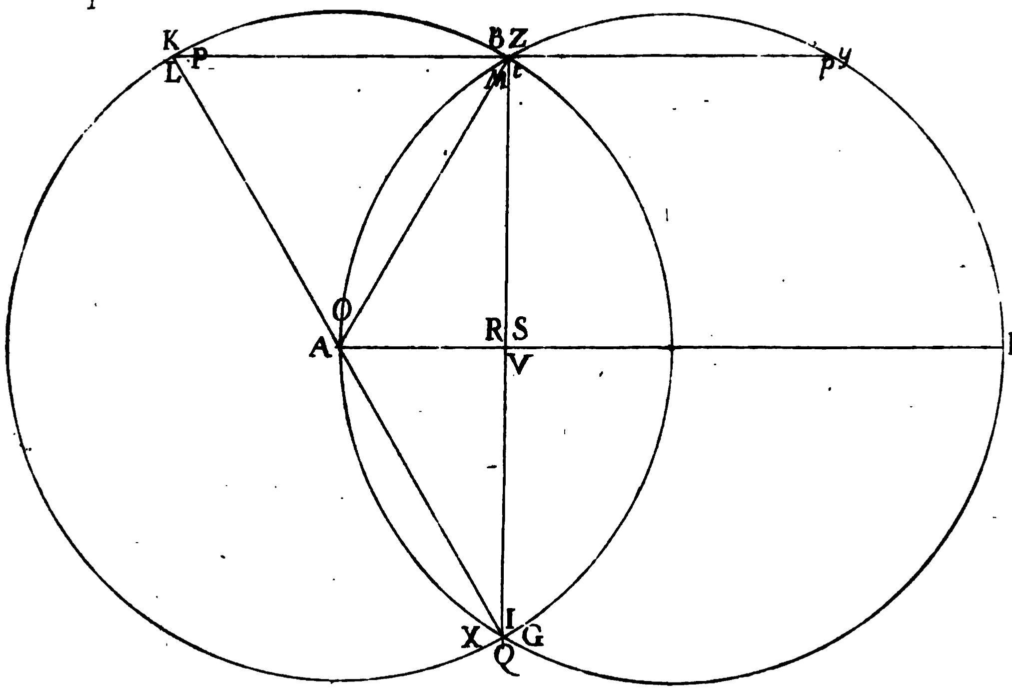


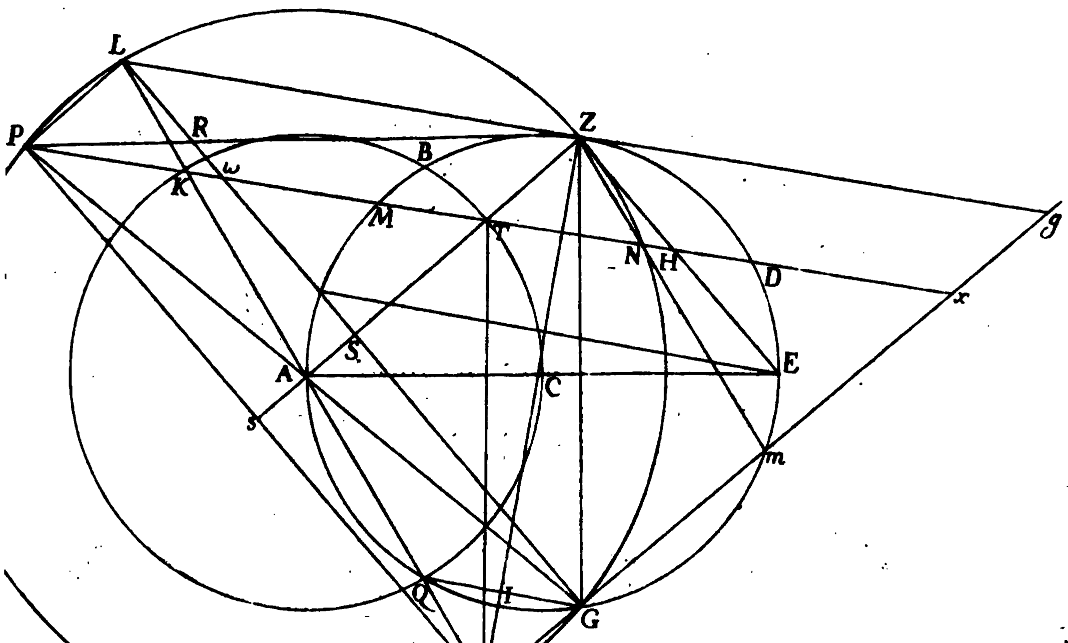
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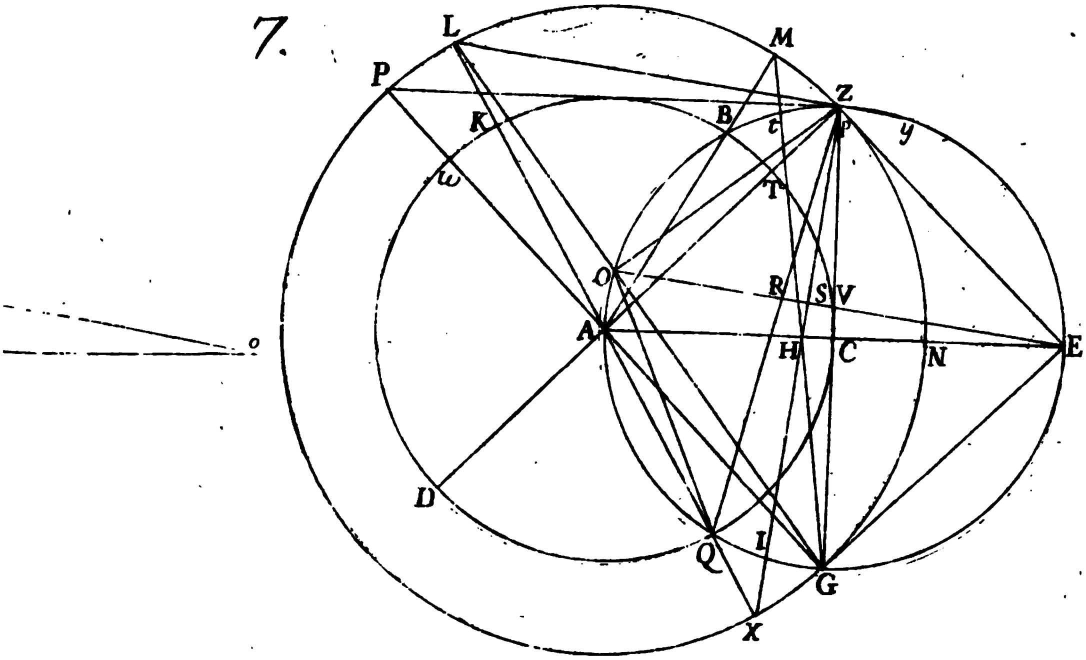
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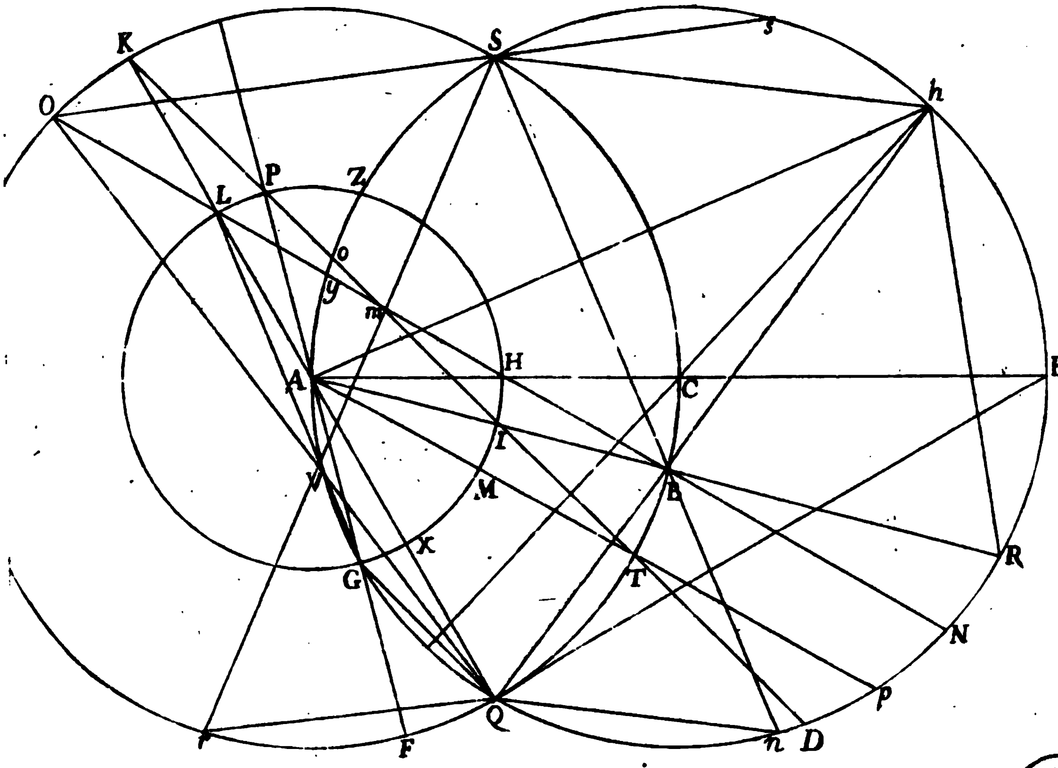


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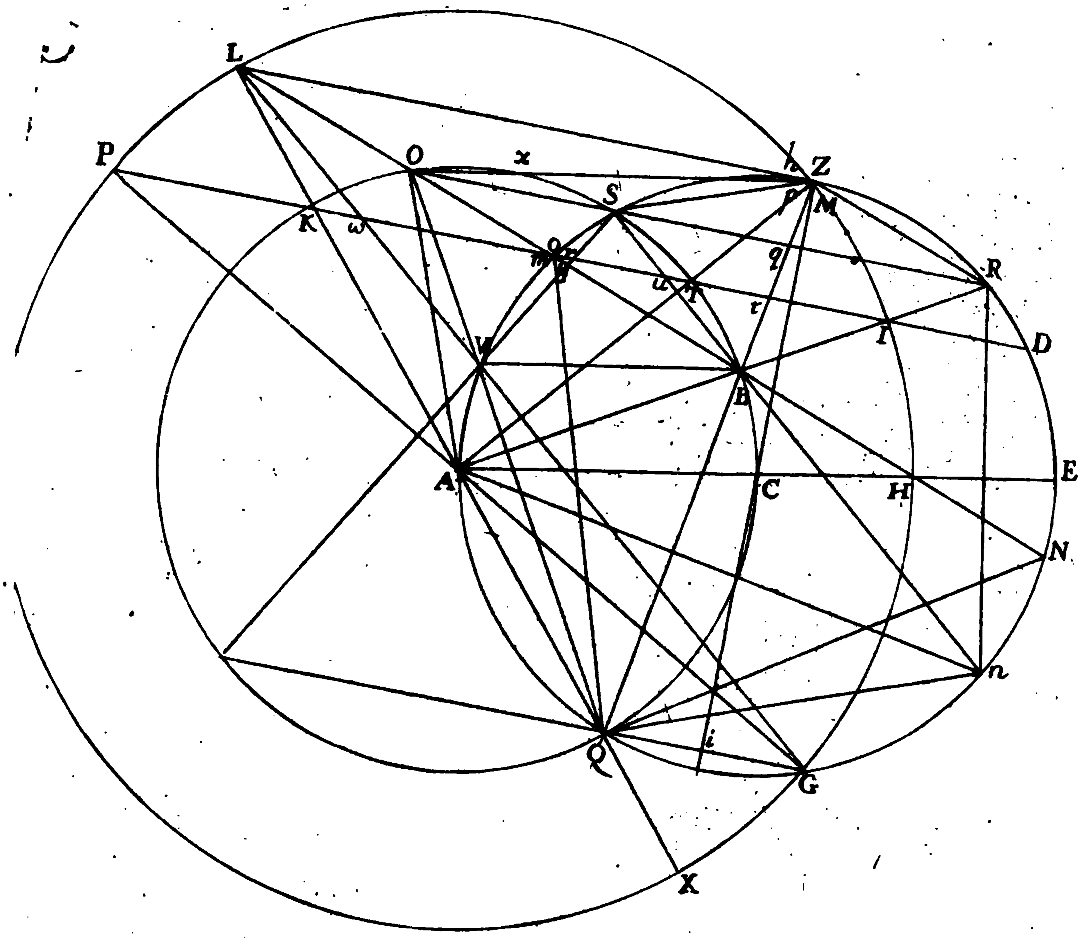
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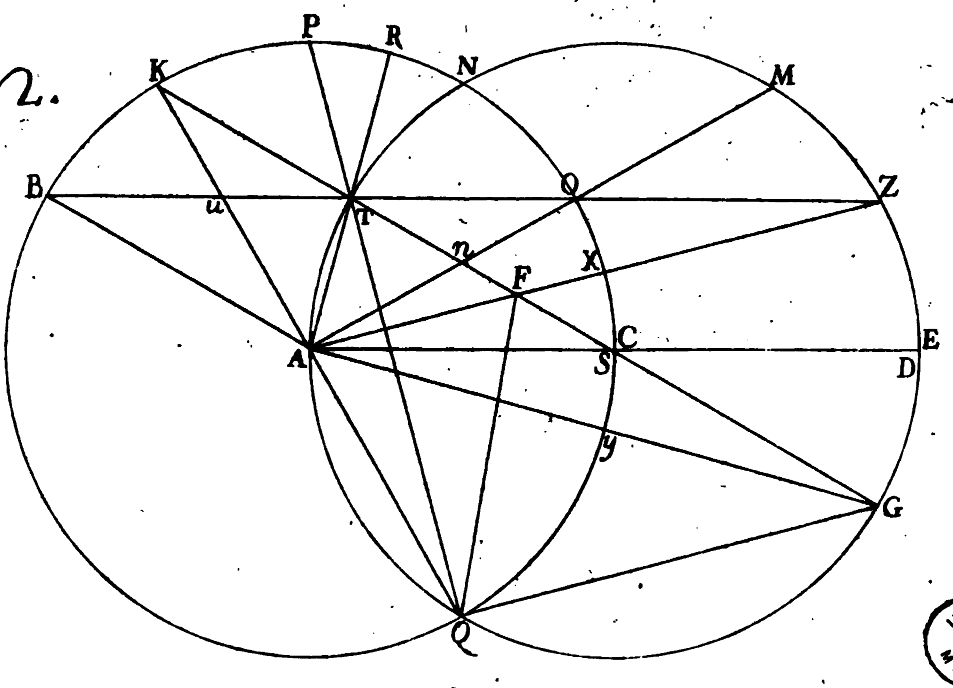


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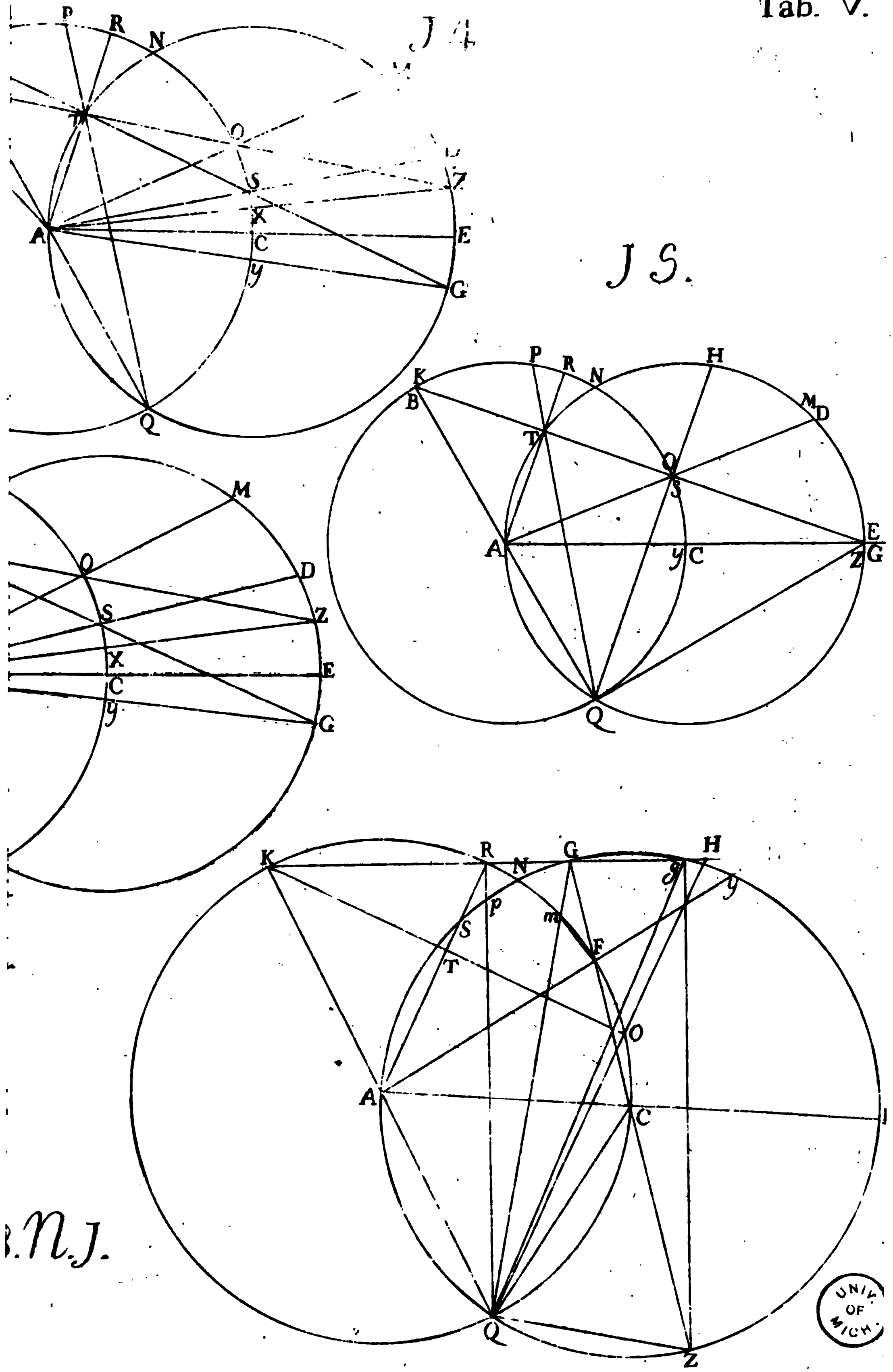


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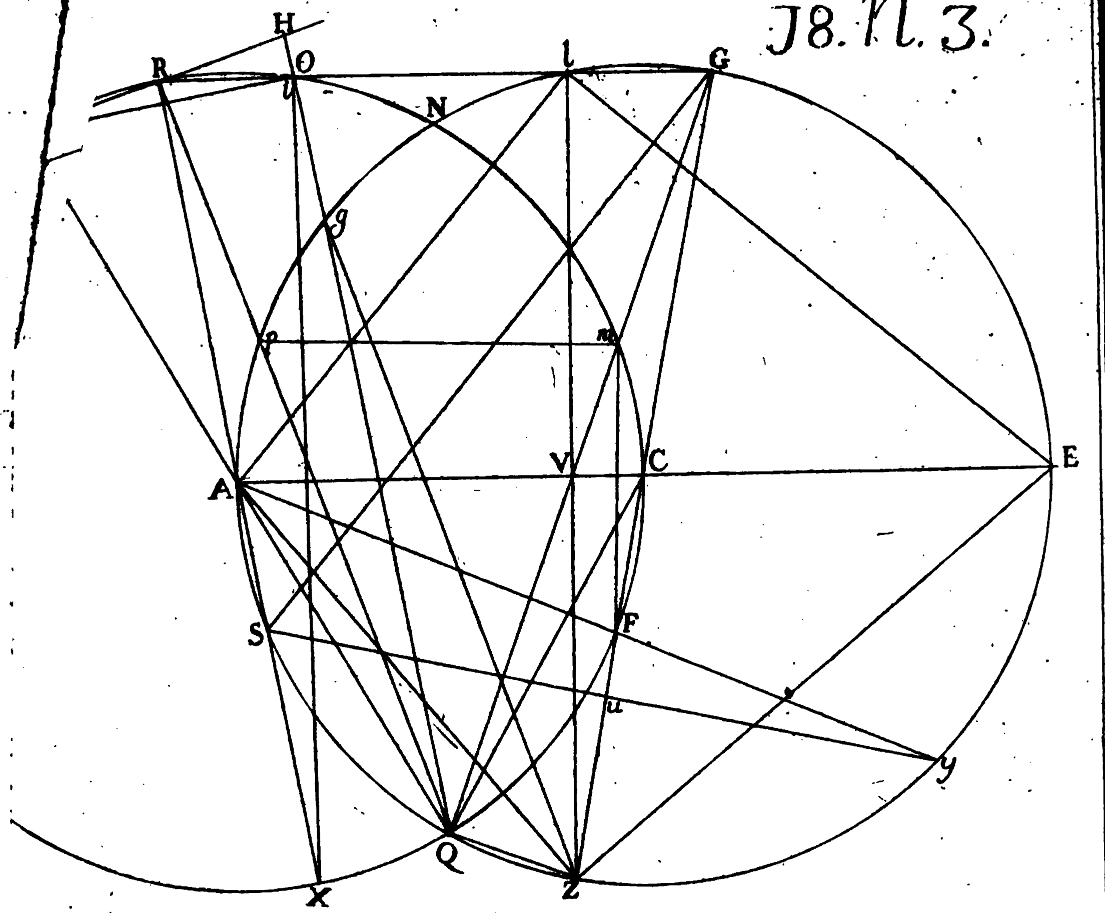

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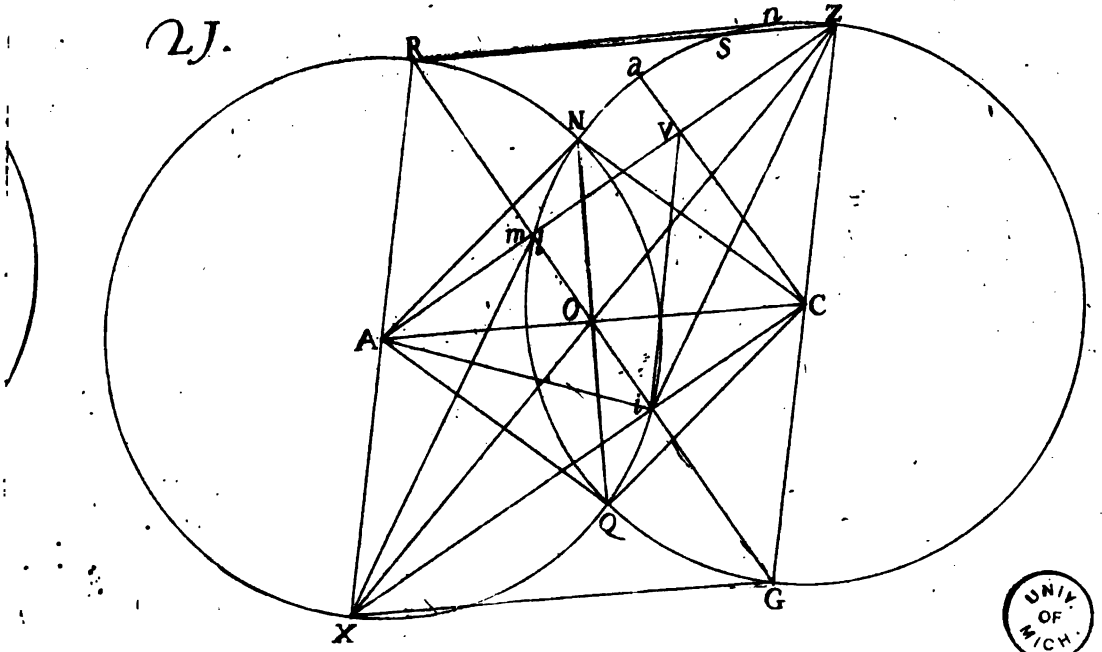


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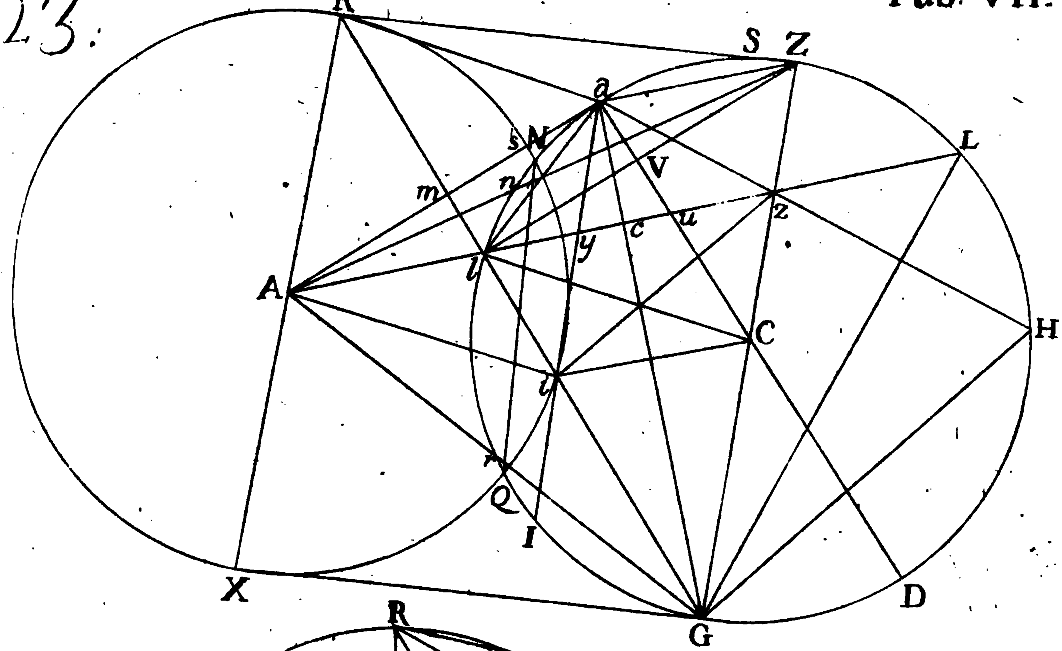


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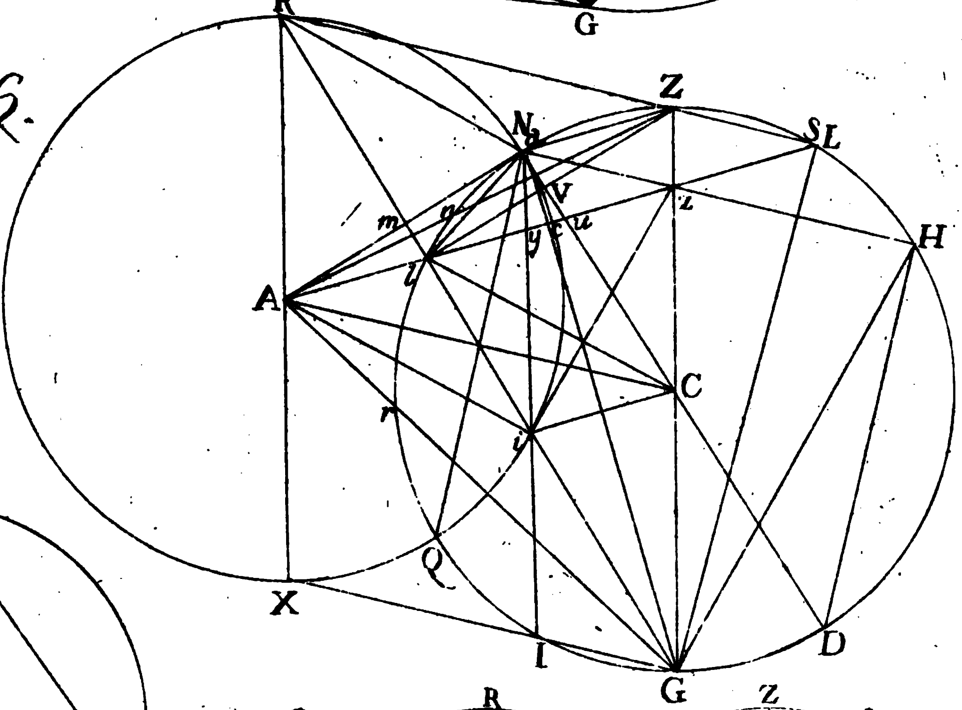


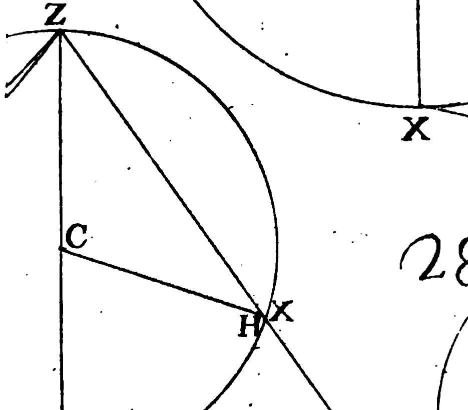
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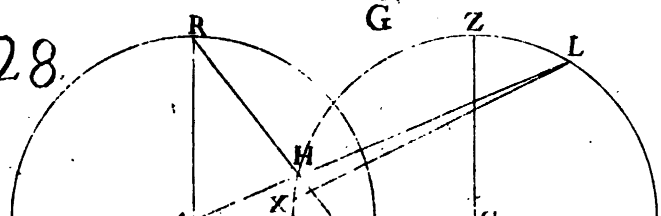

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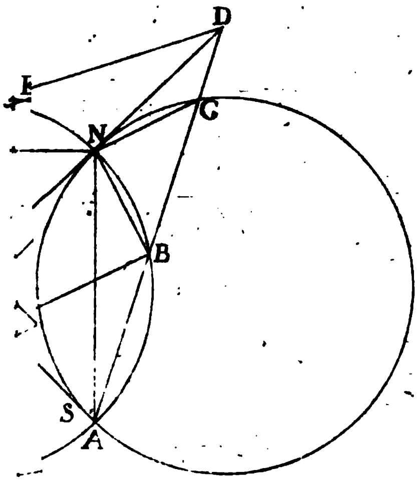
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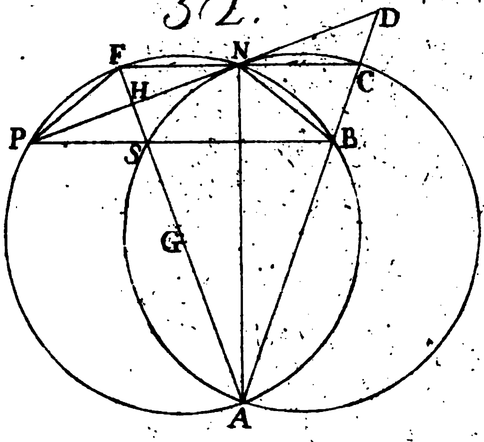

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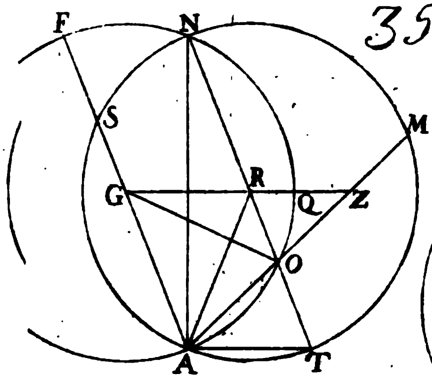

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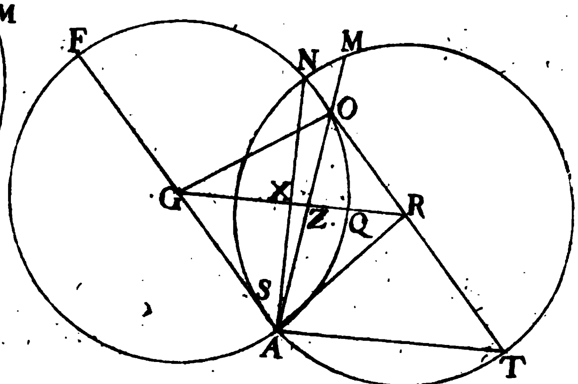
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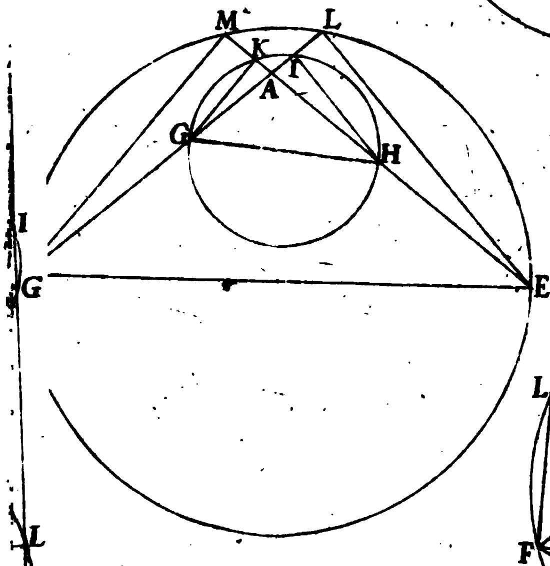

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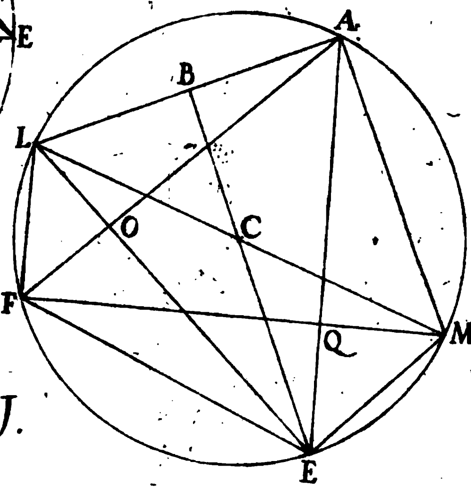

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